relativistic corr.

- $\Delta E_n' = -\frac{\alpha^2 Z^2}{n^2} E_n \left(\frac{3}{4} - \frac{n}{l + \frac{1}{2}} \right)$ relativistic Energy
- $\Delta E'' = -\frac{\alpha^2 Z^2}{n^2} E_n \frac{n \cdot \frac{1}{2} \left(j(j+1) - l(l+1) - s(s+1) \right)}{l \left(l + \frac{1}{2} \right)(l+1)}$ spin-orbit $l \neq 0$
- $\Delta E''' = \frac{\alpha^2 Z^2}{n^2} E_n \cdot n$ $l = 0$, Darwin Term

Sum

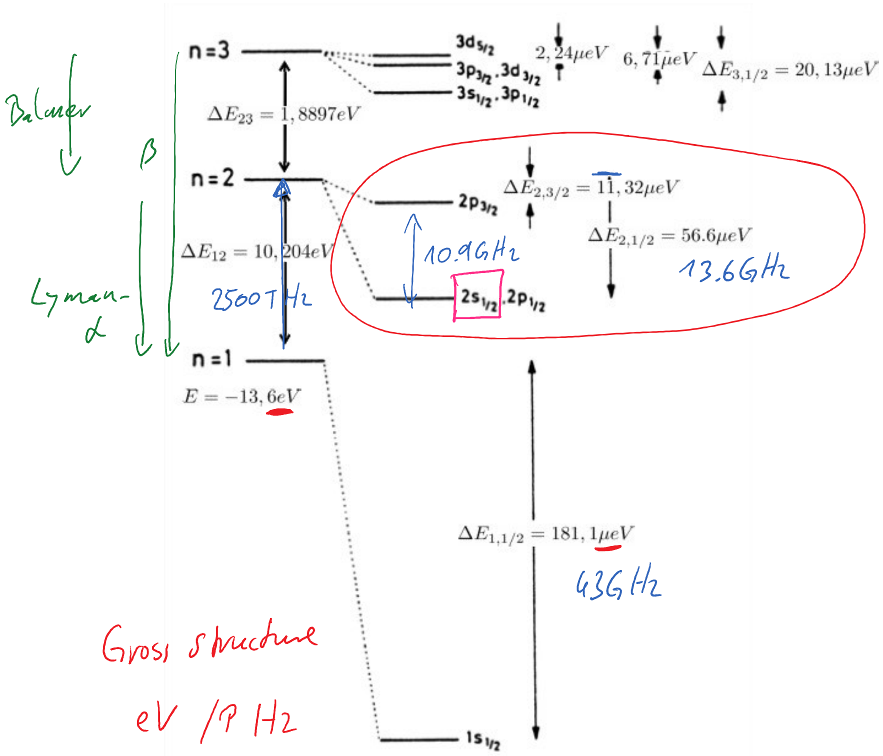
for $l = 0$: $\Delta E_{rel.corr} = \Delta E_n' + \Delta E_n'''$

$l \neq 0$: $\Delta E_n' + \Delta E_n''$

is always $\boxed{\Delta E = -\frac{\alpha^2 Z^2}{n^2} E_n \left(\frac{3}{4} - \frac{n}{j + \frac{1}{2}} \right)}$

↳ known from the Dirac Eqn. Solution ✓

H à la Bohr & Dirac



Fine structure

μeV / GHz

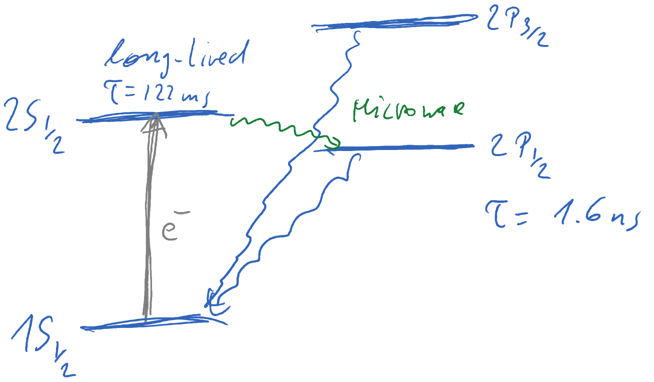
Gross structure

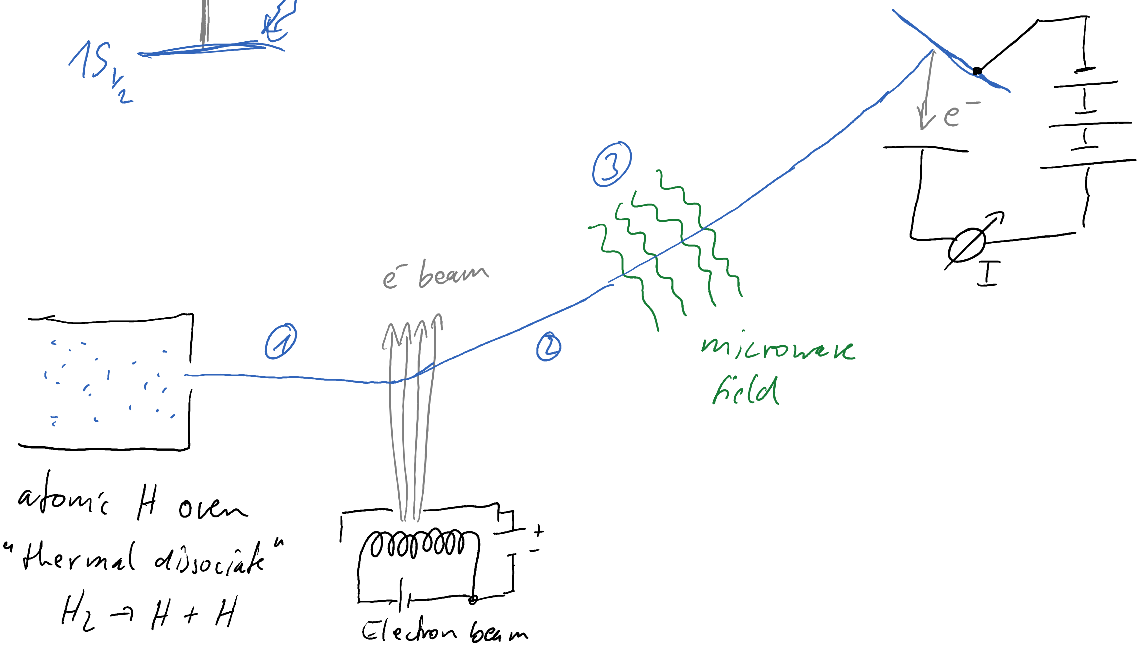
eV / PHz

Lamb shift

Lamb, Retherford 1947-52

idea: use μ waves (RADAR, WW II) to measure the fine structure in H





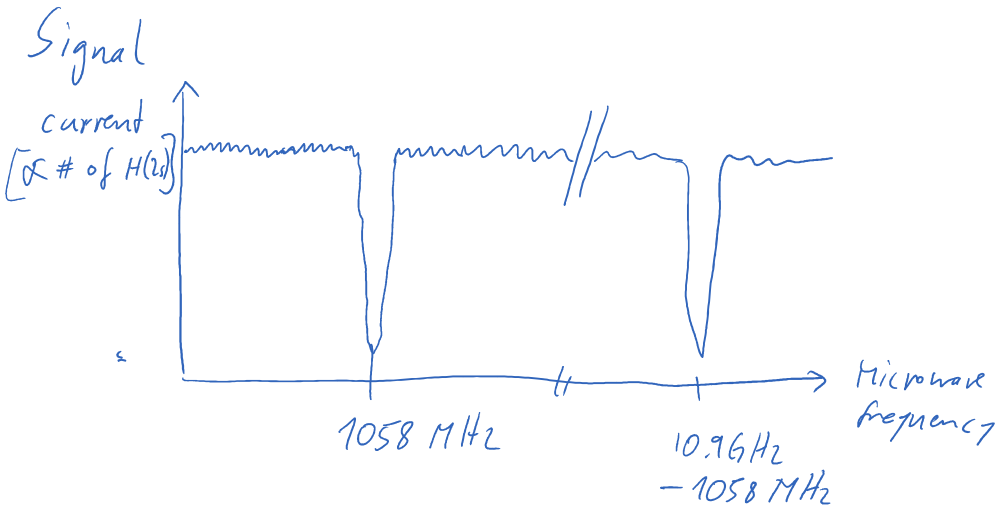
① H beam in $1S_{1/2}$ ground state

② H atom excitation to $2S_{1/2}$
10.2 eV excitation, deflection of H beam
2s state is metastable ($\hat{=}$ long-lived)
(no electric Dipole transition possible to ground state)

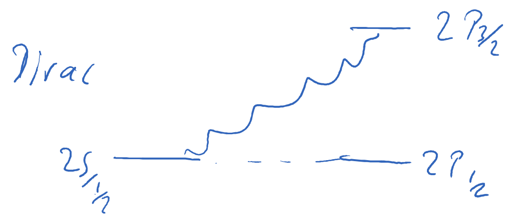
③ Drive $2S_{1/2} \longleftrightarrow 2P$ transition(s) in the microwave field
if successful ($\hat{=}$ microwave frequency matches the energy difference of the levels):

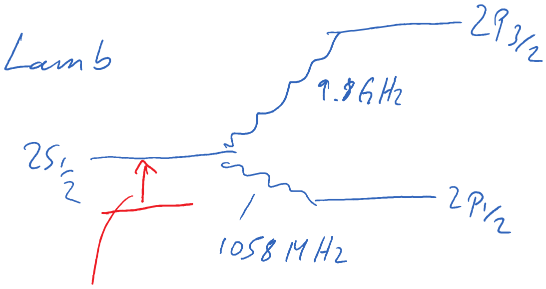
2P states decay immediately (1.6 ns) to the ground state (121.6 nm light) "Lyman-α"

④ The remaining 2S atoms hit a metal plate
10 eV excitation energy produces e^-, leave plate, acceleration
⇒ measure current



"1000 megacycles"



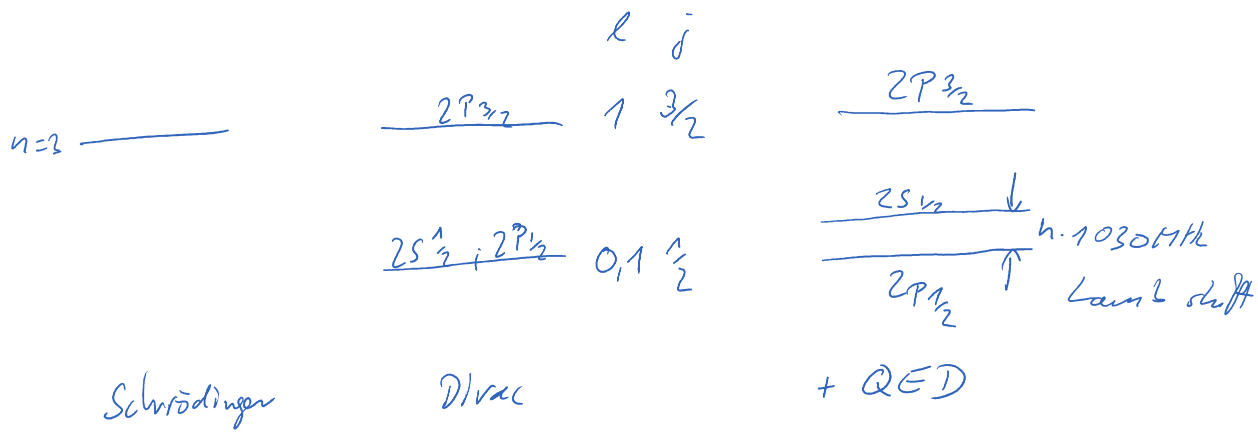


Lamb shift ↳ Dirac Equation

The Lamb shift

l j

$2P_{3/2}$

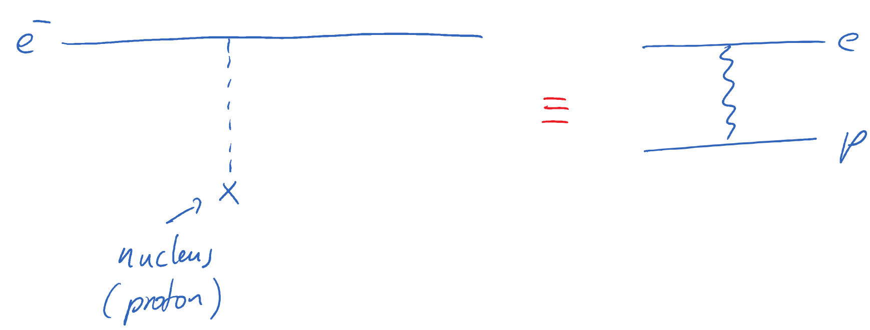


lifts degeneracy in j

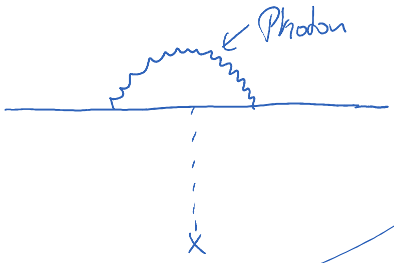
interaction of the H atom with "quantum vacuum"

Feynman diagrams to "visualize" QED effects

leading order



NLO

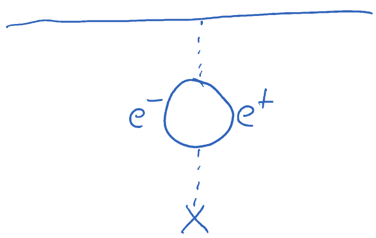


"Self-energy" of e^-

e^- is "smeared out" because it is lacking energy/momentum

+1060 MHz

"Zitterbewegung"



"Vacuum polarization"

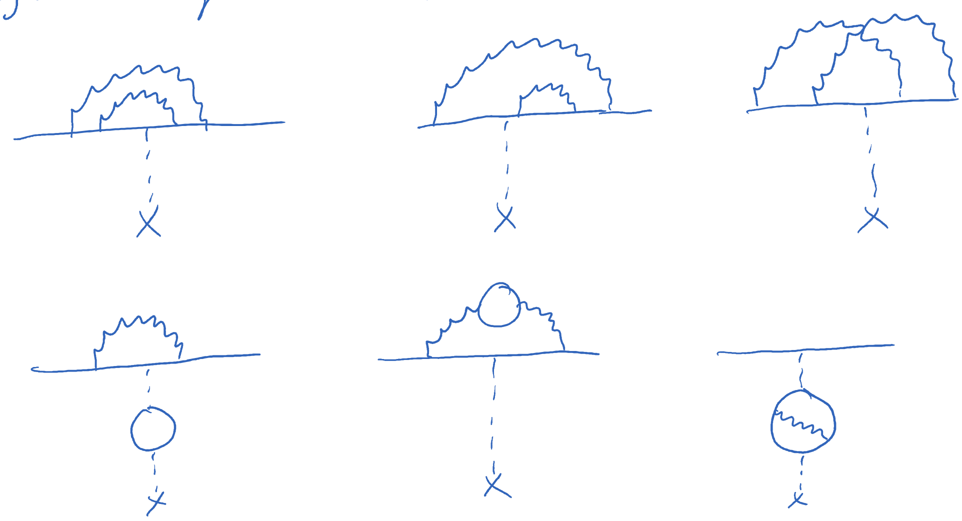
− 30 MHz

Bethe: "Uehling's correction is

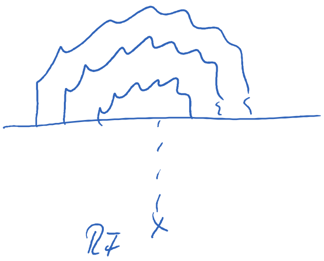
too small & has the wrong sign"

and higher order corrections

e.g. "2 loop corrections"



QED for $1\, e^-$ "g-2 of the electron"



5 loops

. . . .

$$g = \underline{2 + \frac{1}{2}\frac{\alpha}{\pi}} + c_2\left(\frac{\alpha}{\pi}\right)^2 + c_3\left(\frac{\alpha}{\pi}\right)^3 \ldots$$

Julian Schwinger 1948

$g-2$ of e^- + calculations $\Rightarrow \alpha$ (fine structure constant)
