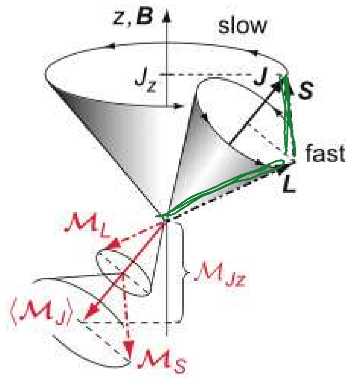
Hyperfine Structure (HFS)

Mittwoch, 17. November 2021 14:11

Hertel, Schulz:
Atoms, Molecules and Optical Physics 1

so far: $\vec{J} = \vec{L} + \vec{S}$



But nuclei have spins, too!

HFS is result of coupling of nuclear spin to the electron cloud

Nuclear spin: I e.g. $I(\text{proton}) = \frac{1}{2}$

associated magnetic moment $\hat{\vec{M}} = g_I \mu_N \hat{\vec{I}}/\hbar$

z component $\hat{M}_z = g_I \mu_N \hat{I}_z/\hbar$

μ_N = nuclear magneton

$$\mu_N = \frac{e\hbar}{2 m_p} = \frac{m_e}{m_p} \mu_B$$

$\hookrightarrow \frac{1}{2000} \Rightarrow$ HFS is $O(1000\times)$ smaller than FS

$$\mu_N = 5.05 \cdot 10^{-27}\, J/T = 3.15 \cdot 10^{-8}\, eV/T$$

$$\hat{=}\ 7.623\ MHz/T$$

cf. Bohr magneton (magn. moment of e^-)

$$\mu_B = 5.79 \cdot 10^{-5}\ \mathrm{eV/T} \ \hat{=}\ 14\ \mathrm{GHz/T}$$

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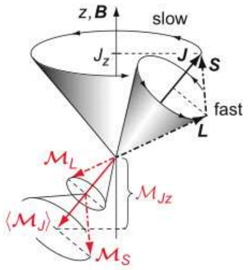
let's couple!

so far L, S, J $\longrightarrow$



$\downarrow \quad \downarrow \quad \downarrow$

$J \quad I \quad F$

$$|\vec{F}| = \sqrt{F(F+1)} > F$$

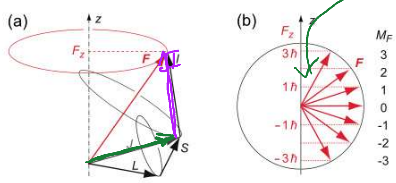


Fig. 9.1 (a) Vector model for coupling L and S to J, and of J and I to F. (b) F has $2F+1$ possibilities of orientation in space

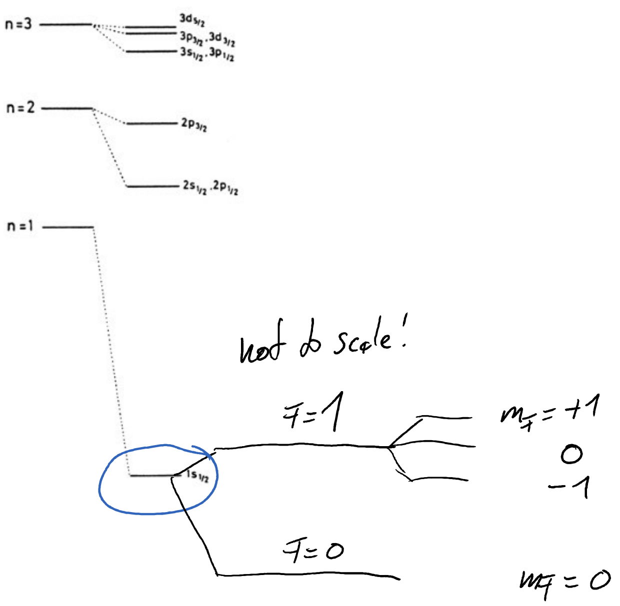
e.g. $F = 3$

$m_F = -3, \dots +3$

e.g. hydrogen

$m_F = -F, \dots F$

Schrödinger Dirac



1S ground state

$L = 0, \ S = \frac{1}{2}$

$\Rightarrow J = \frac{1}{2}$

proton $I = \frac{1}{2}$

$\vec{F} = \vec{I} + \vec{J}$

$$\rightarrow F = \begin{cases} \frac{1}{2} + \frac{1}{2} = 1 & \uparrow\uparrow \\ \frac{1}{2} - \frac{1}{2} = 0 & \uparrow\downarrow \end{cases}$$

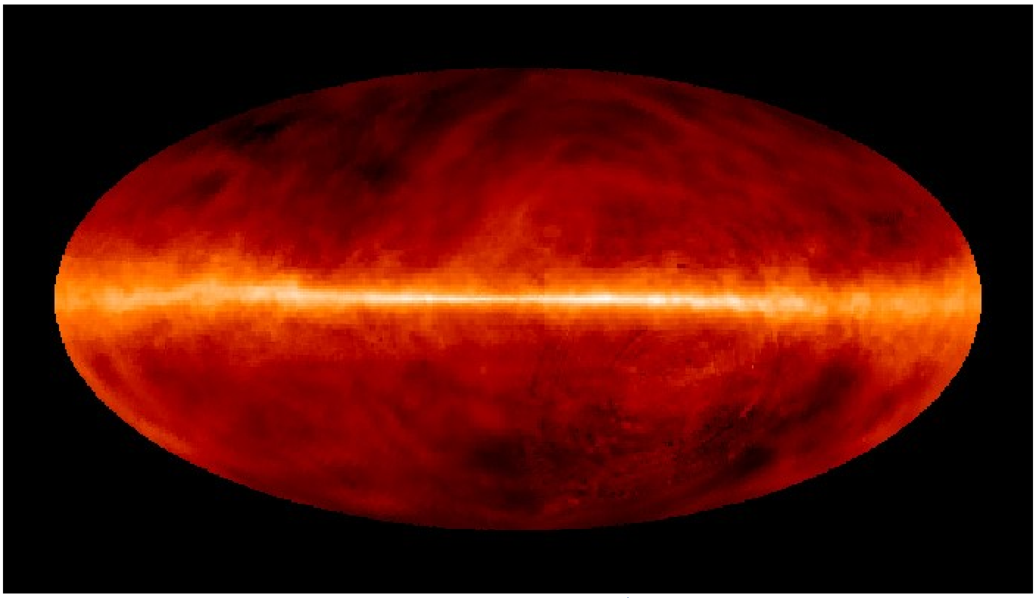
e^- spin, p spin

..

1s ground state hyperfine splitting in hydrogen

"21cm line" = wavelength

$\hat{=}$ 1.4 GHz = frequency



Milky way in 21cm "light"

- also: hydrogen MASER → frequency standard

Microwave → Light
Amplification by
Stimulated
Emission of
Radiation

F=1 $\uparrow\uparrow$ → $\uparrow\downarrow$ F=0 + 1 photon @ 1.4 GHz

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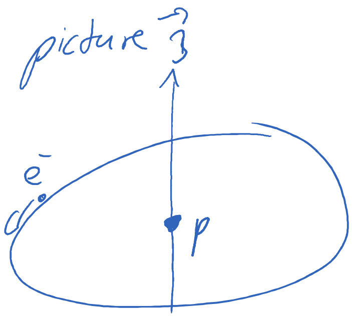
classical picture



e^- orbits nucleus $\hat{=}$ circular current

$\Rightarrow$ magnetic field

$\propto L$

nucleus sits inside this magnetic field produced by the e^-

$$H_{HFS} = \vec{\mu}_N \cdot \vec{B}_{e^- \text{cloud}}$$

$$= g_N \, \mu_N \, \frac{\vec{I}}{\hbar} \cdot B_0 \frac{\vec{J}}{|\vec{J}|}$$

$\hookrightarrow$ $g_{proton} \approx 5.58 \ldots$ $\quad \neq 2$ $\quad$ $p \neq$ elementary particle

is not point-like

$$g_{proton} = 2.792\,847\,350\,(7)(6) \cdot 2$$

$\hookrightarrow$ J. Walz @ JGU

A. Mooser et al Nature 509, 596 (2014)

$\pm\, 0.000\,000\,007$ | stat

$\pm\, 0.000\,000\,006$ | syst

even better: Schneider et al (2017)

$3.14157\,(15)(1)$

$\pm\, 0.00\ \ 15$

$\pm\ \ .00\ \ 01$

$$|\vec{J}| = \hbar \sqrt{J(J+1)}$$

$$H_{HFS} = \frac{g_N \, \mu_N \, B_0}{\hbar^2 \sqrt{J(J+1)}} \cdot \vec{I} \cdot \vec{J} \equiv A \cdot \vec{I} \cdot \vec{J}$$

Hyperfine A constant

$$\vec{F} = \vec{I} + \vec{J}$$

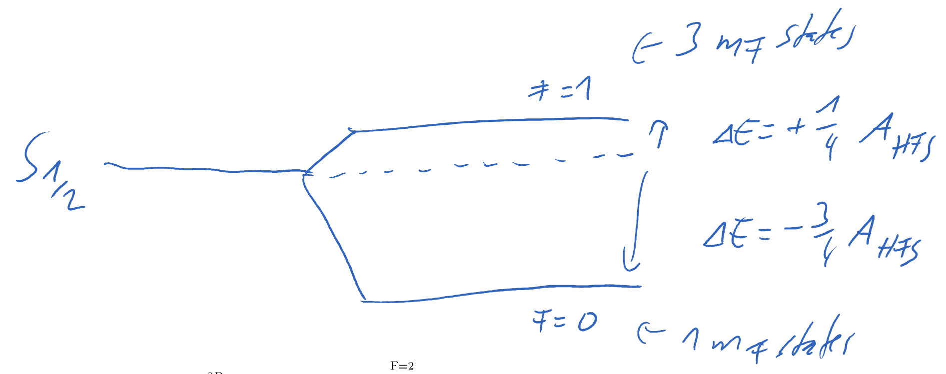
$$\vec{F}^2 = \vec{I}^2 + \vec{J}^2 + 2\vec{I}\vec{J}$$

$$\Delta E \ldots = \frac{A_{HFS}}{\ } \left[F(F+1) - I(I+1) - J(J+1) \right]$$

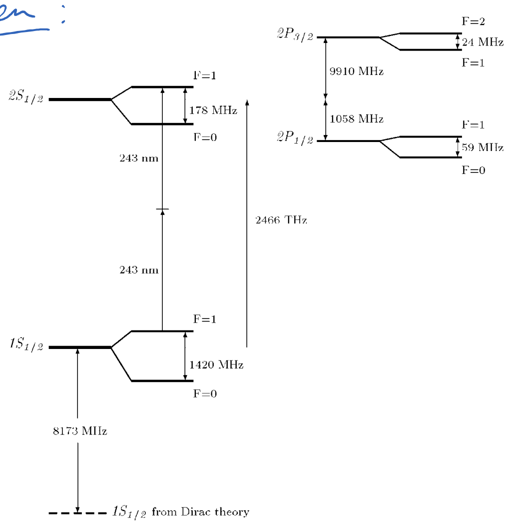
$$\Delta E_{HFS} = \frac{A_{HFS}}{2}\left[F(F+1) - I(I+1) - J(J+1)\right]$$

A_{HFS} must be taken from experiment

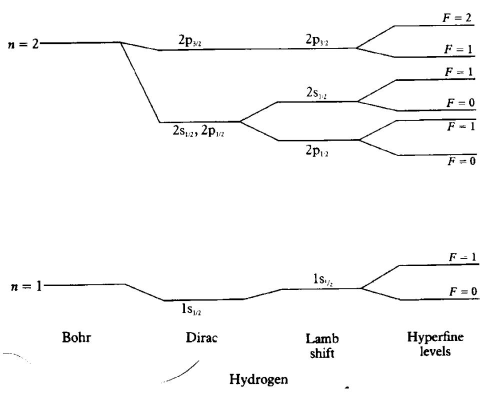
Hydrogen



Hydrogen :



Hydrogen $I = \frac{1}{2}$



Deuterium $I = 1$