Ex5a

Exercise 4

1 Spin-orbit interaction

The spin-orbit coupling of the angular momentum $\vec{l}$ and the spin $\vec{s}$ of the electron leads to the total angular momentum $\vec{j}$.

(a) Write down the spectroscopic notation (analogue to e.g. $2s_{1/2}$) for all possible states with $n = 1,\ 2,\ 3$ in the hydrogen atom.

(b) Show that the fine structure splitting ΔE_{ls} between two adjacent levels with $j = \ell \pm 1/2$ is proportional to

$$\Delta E_{ls} \propto \frac{Z^4}{n^3 \ell(\ell+1)}.$$

Hint: The energy levels including spin-orbit coupling can be written as

$$E_{nlj} = E_n + A \langle \vec{s} \cdot \vec{\ell} \rangle \qquad \text{with} \qquad A = -E_n \frac{Z^2 \alpha^2}{n \cdot \ell(\ell+1/2)(\ell+1)},$$

where E_n denote the Bohr energy levels.

(c) Calculate the level shift (in eV) caused by the spin-orbit coupling for the 2p levels in hydrogen.

(d) Calculate the level shift (in eV) caused by the spin-orbit coupling of the 2p levels in boron-like Argon (Ar^{13+}), a system with 5 electrons (a single electron in the 2p level). Assume that the 2p electron sees an effective nuclear charge of $Z = 15.78$. What is the corresponding wavelength for transitions between the 2p states?

2 Stern-Gerlach experiment

The Stern-Gerlach experiment demomonstrated the quantization of magnetic moments in atoms.

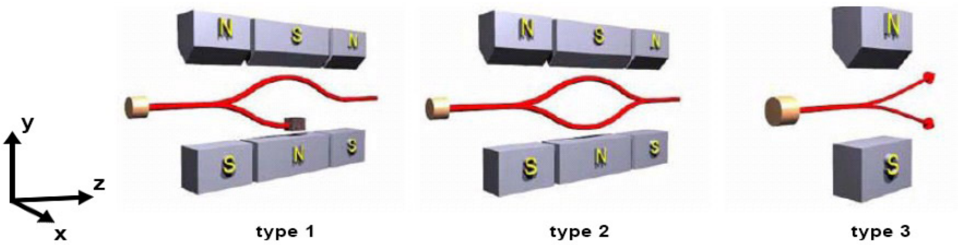
(a) Let's consider a beam of silver atoms (mass $M = 108\,\mathrm{u}$) along the y-axis, that is created inside an oven heated to a temperature of about T=1000 °C. The magnetic moment of the atoms is created by a single electron with spin $s = 1/2$. The beam passes an apparatus of length $L_1 = 1\,\mathrm{m}$. Inside the apparatus, the atoms experience an inhomogeneous magnetic field of $\frac{\partial B}{\partial z} = 10\,\mathrm{T\,m^{-1}}$ that is aligned perpendicular to the atoms motion. The position of the atoms is recorded with a detection screen at a distance of $L_2 = 1\,\mathrm{m}$ behind the apparatus.

(i) Calculate the spatial splitting of the atoms with $v_{y,0} = \sqrt{3k_BT/M}$ (most probably velocity for a thermal atomic beam) on the screen.

(ii) Assume a symmetric distribution of velocities v_y with a full-width-at-half-maximum given by $\sqrt{2k_BT/M}$. What is the corresponding spread of the detected atoms on the screen? Is it possible to observe the splitting of the spin states on the screen?

(iii) Explain if it is possible to use this Stern-Gerlach magnet to create a spin-polarized electron beam?

(b) In the following, three different types of Stern-Gerlach magnets are displayed. The atom beam (red lines) consists of unpolarized alkali atoms, which have a single valence electron with $j = 1/2$. The beam propagates along the z-direction. In each case, explain what can be observed on the screen.



(I) Let's consider a Stern-Gerlach experiment where the $j_y = -1/2$ component is blocked by a metal piece (type 1 magnet aligned to the y-axis). Subsequently, the beam passes a magnet of type 3, that is oriented in the same direction.

(II) Same configuration as in (I), but now the type 3 magnet is rotated by 90°.

(III) The third experiment consists of a type 1 magnet along the y-axis, followed by a rotated type 2 magnet along the x-axis and a type 3 along the y-axis. What changes if we replace the second element by a magnet of type 1 with same orientation?

3 Measurement of the Lamb shift in hydrogen

The Lamb shift of the 1s ground state in atomic hydrogen L_{1s} was determined by means of high-precision laser spectroscopy[1]. L_{1s} was extracted from the combined energy differences between the 1s, 2s and 4s states, namely

$$\Delta E = (E_{4s} - E_{2s}) - \frac{1}{4}(E_{2s} - E_{1s}).$$

For simplicity, we neglect the hyperfine-structure in hydrogen. Due to the $1/n^2$-dependency of the binding energy (Bohr formula), non-relativistic contributions to ΔE cancels. Thus, ΔE only contains Dirac and QED (Lamb shift) contributions.

In the experiment, $\Delta E/h$ was measured to be $4797\,\mathrm{MHz}$. Based on this result, calculate the 1s Lamb shift L_{1s} in atomic hydrogen (in MHz).

Note: The 2s Lamb shift $L_{2s} = 1057\,\mathrm{MHz}$ (measured by Lamb and Retherford) and the 4s Lamb shift $L_{4s} = 131.66\,\mathrm{MHz}$ can be considered to be known as well as the formula for the Dirac energy levels (given in the lecture).

[1] M. Weitz *et al.*, Phys. Rev. Lett. **72**, 328 (1994)