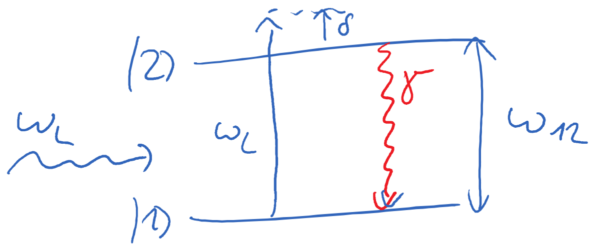
Density matrix

Donnerstag, 9. Dezember 2021 13:31

SE is always coherent

but we want spontaneous decay



before: pure states

$$\Psi(\vec{r},t) = c_1 e^{-i\omega_1 t} u_1(\vec{r}) + c_2 e^{-i\omega_2 t} u_2(\vec{r})$$

coherent superpos.

$|c_2|^2$ is described by SE

density matrix operator

system with several states

probability (statistical sense) to find atom in state $|\Psi_k\rangle$ is p_k

$$\hat{\rho} = \sum p_k |\Psi_k\rangle\langle\Psi_k|$$

elements of DM

$$\rho_{ij} = \langle i|\hat{\rho}|j\rangle$$

DM can describe states as statistical

mixtures

e.g. our 2-level system

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \begin{pmatrix} c_1 c_1^* & c_1 c_2^* \\ c_2 c_1^* & c_2 c_2^* \end{pmatrix}$$

populations
$\hat{=}$ probability to
find atom in state $|1\rangle$ or $|2\rangle$

coherences
relate phase between states
if $\rho_{ik} = 0 \rightarrow$ phase between $|i\rangle$ & $|k\rangle$ is undefined

Example: $c_1 = c_2 = \frac{1}{\sqrt{2}} \rightarrow \rho = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

stat. probab. $= \frac{1}{2}$ (diagonal el.)

coherences $\neq 0 \Rightarrow$ well-defined phase
↳ constant in time

$$\Psi(\vec{r},t) = |c_1| e^{-i\omega_1 t} u_1(\vec{r}) + |c_2| e^{i\varphi} e^{-i\omega_2 t} u_2(\vec{r})$$

Ex. 2: completely incoherent state
↳ φ fluctuates between $0, 2\pi$
every measurement gives different result

off-diag. el. $= 0$

$$\rho_{12} = \langle |c_1| |c_2| e^{-i\varphi} \rangle_{0,2\pi} = 0$$

$\rightarrow$ DM $\quad \rho = \begin{pmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

50% in $|1\rangle$, $|2\rangle$

will get us to ....