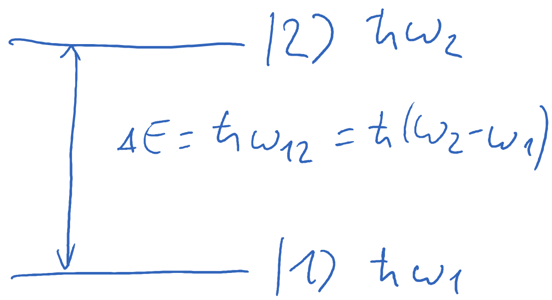
Light-atom interaction

$$H = H_{atom} + V(t)$$

- 2 level system



light field

(angular) frequency

ω_L

- interaction $\quad V = \vec{d} \cdot \vec{E}(\vec{r}, t)$

dipole operator
atomic dipole moment

electric field of the light

- semiclassical approx: light field not quantized

- "Dipole approximation"
  - atom size $\sim a_0 \sim 0.05\,nm$
  - wavelength of light $\sim$ size over which the electric field changes $\sim$ few 100 nm

  ↳ neglect variation of field over size of atom

- atom does not change its position during the interaction

$$\vec{E}(\vec{r}, t) \overset{(1)}{=} \vec{E}(\vec{R}, t) \overset{(2)}{=} E(t) = \vec{\varepsilon} E_0 \cos(\omega_L t)$$

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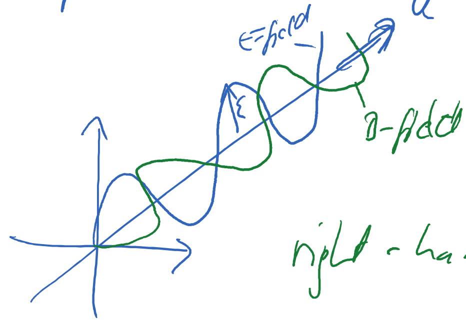
atom position

polarization vector $\vec{k}$



right-handed

$$V = -\vec{d} \cdot \vec{E}(t) \quad \text{with } \vec{d} = -e\vec{r}$$

time dependent SE

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{r},t) = H \Psi(\vec{r},t) = (H_0 + V(t)) \Psi(\vec{r},t)$$

ansatz:

$$\Psi(\vec{r},t) = c_1(t) e^{-i\omega_1 t} u_1(\vec{r}) + c_2(t) e^{-i\omega_2 t} u_2(\vec{r})$$

time-dependent amplitudes

= time evolution without electric field

into SE

$$\dot{c}_1(t) = i \frac{d_{12}^{\varepsilon} E_0}{\hbar} e^{-i\omega_{12} t} \cos(\omega_L t)\, c_2(t)$$

$$\dot{c}_2(t) = i \frac{d_{12}^{\varepsilon} E_0}{\hbar} e^{+i\omega_{12} t} \cos(\omega_L t)\, c_1(t)$$

$$d_{12}^{\varepsilon} = \langle 1 | \hat{\vec{d}} | 2 \rangle \cdot \vec{\varepsilon} \overset{\text{abbrev}}{=} d$$

projection of dipole matrix element
onto polarization vector

d gives strength of transition

$\hat{=}$ $d = 0 \rightarrow$ forbidden transition

<u>important quantity</u>

resonant Rabi frequency: $\Omega_0 = \frac{E_0 \cdot d}{\hbar}$

$$\rightarrow \dot{c}_1(t) = i\,\Omega_0\, e^{-i\omega_{12}t} \cos(\omega_L t)\; c_2(t)$$

$$\dot{c}_2(t) = i\,\Omega_0\, e^{+i\omega_{12}t} \cos(\omega_L t)\; c_1(t)$$

$$\cos(\omega_L t) = \frac{1}{2}\left(e^{i\omega_L t} + e^{-i\omega_L t}\right) \quad \text{Euler}$$

$$\Rightarrow \dot{c}_1(t) = i\,\frac{\Omega_0}{2}\, e^{\underline{-i\omega_{12}t}} \left(e^{\underline{i\omega_L t}} + e^{-i\omega_L t}\right) c_2(t)$$

$$c_2 \quad \ldots \quad c_1(t)$$

we only consider near-resonant coupling

$\downarrow$

$$\omega_L \approx \omega_{12}$$

slowly oscillating $e^{-i(\omega_{12}-\omega_L)t}$

fast oscillating $e^{-i(\omega_{12}+\omega_L)t}$

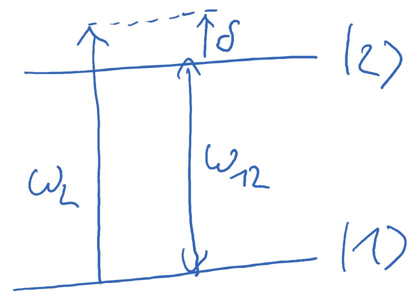
$\hookrightarrow$ oscillate with $O(1000\,\text{THz})$

average quickly

↳ can be neglected

↳ "Rotating wave approximation" (RWA)

Detuning $\delta = \omega_L - \omega_{12}$



$$\dot{c}_1(t) = i\frac{\Omega_0}{2} e^{i\delta t} c_2(t)$$
$$\dot{c}_2(t) = i\frac{\Omega_0}{2} e^{-i\delta t} c_1(t)$$

go into rotating frame @ ω_L

new coefficients

$$\tilde{c}_1(t) = c_1(t) \cdot e^{-i\frac{\delta}{2}t}$$
$$\tilde{c}_2(t) = c_2(t)\, e^{+i\frac{\delta}{2}t}$$

→ system of coupled differential equations

$$\frac{d}{dt}\begin{pmatrix} \tilde{c}_1(t) \\ \tilde{c}_2(t) \end{pmatrix} = \frac{i}{2}\begin{pmatrix} -\delta & \Omega_0 \\ \Omega_0 & \delta \end{pmatrix}\begin{pmatrix} \tilde{c}_1(t) \\ \tilde{c}_2(t) \end{pmatrix}$$

simplest case : $\boxed{\delta = 0}$ resonant light-atom interaction

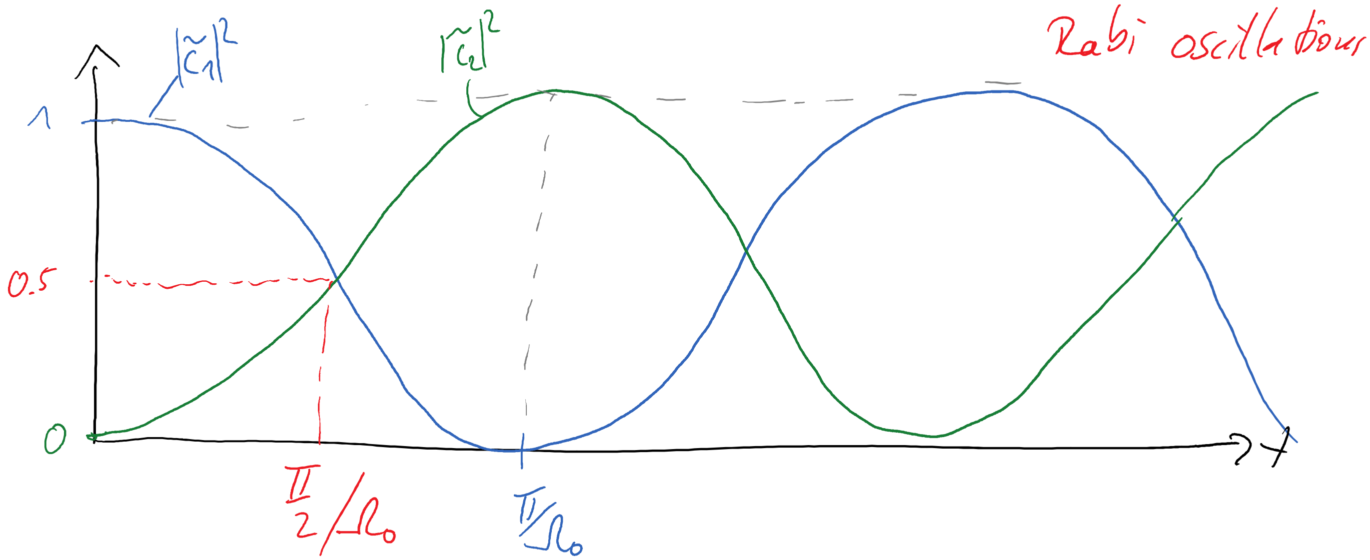
$$\Rightarrow \ddot{\tilde{c}}_1(t) = -\frac{\Omega_0^2}{4}\tilde{c}_1(t)$$

e.g. initially atom in g.s. $|1\rangle$

$$\tilde{c}_1(t=0) = 1$$
$$\tilde{c}_2(t=0) = 0$$

solution
$$\tilde{c}_1(t) = \cos\left(\frac{\Omega_0}{2}t\right)$$
$$\tilde{c}_2(t) = i\sin\left(\frac{\Omega_0}{2}t\right)$$

population
$$|\tilde{c}_1(t)|^2 = \cos^2\left(\frac{\Omega_0}{2}t\right) = \frac{1}{2}\left(1+\cos(\Omega_0 t)\right)$$
$$|\tilde{c}_2(t)|^2 = \sin^2\left(\frac{\Omega_0}{2}t\right) = \frac{1}{2}\left(1-\cos(\Omega_0 t)\right)$$



- population oscillates between $|1\rangle$, $|2\rangle$ with resonant Rabi frequency Ω_0
  $\hookrightarrow \delta = 0$ now
- increasing light intensity $\rightarrow$ Ω_0 gets larger

→ faster oscillations

$\frac{\pi}{2}$ pulse : coherent superposition of $|1\rangle$ & $|2\rangle$

π pulse : swaps population

2nd case: $\delta \neq 0$ (laser detuned)

start in g.s. $|1\rangle$

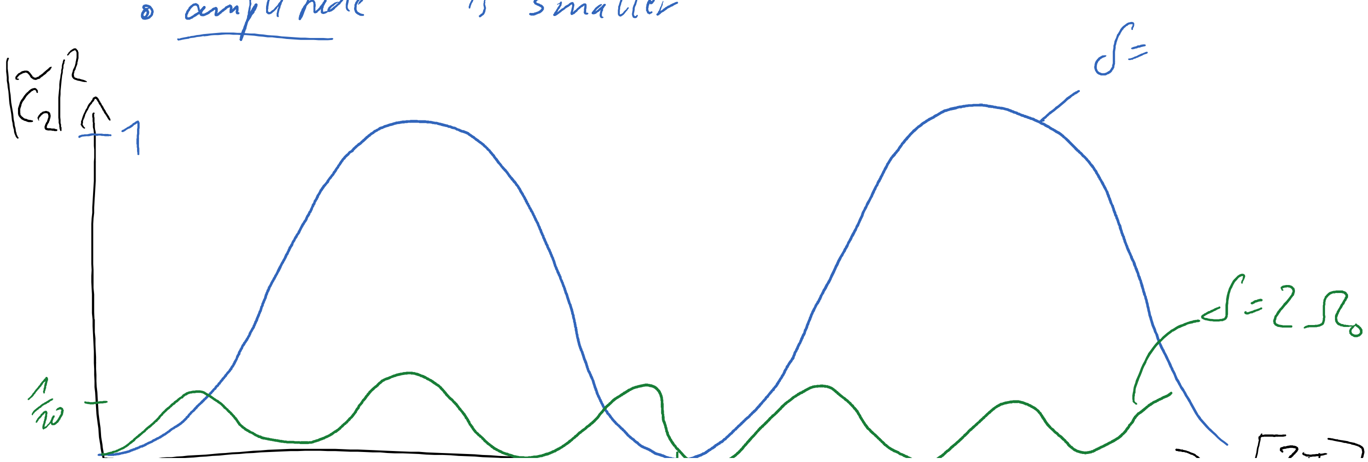
$$|\tilde{c}_2(t)| = \frac{\Omega_0^2}{\Omega^2} \sin^2\left(\frac{\Omega}{2} t\right) =$$

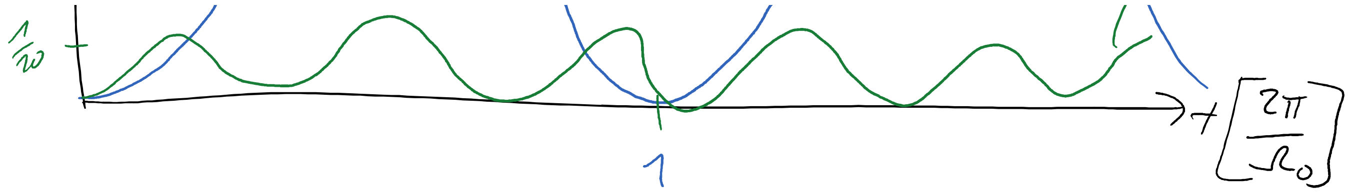
$$= \frac{\Omega_0^2}{2\Omega^2} \left\{ 1 - \cos(\Omega t) \right\}$$

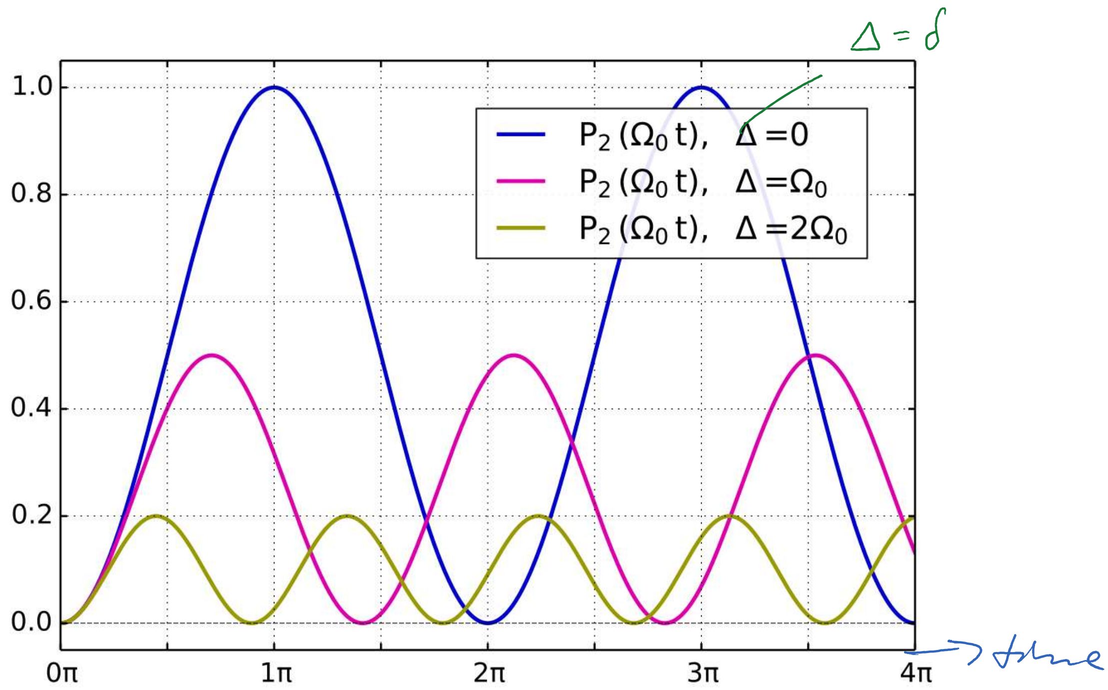
Ω = generalized Rabi frequency

$$\Omega = \sqrt{\Omega_0^2 + \delta^2}$$

- $\Omega \geq \Omega_0$
  - frequency of population oscillation is faster
  - amplitude is smaller




1/20
1
t [2π/Ω_0]


Δ = δ
1.0
0.8
0.6
0.4
0.2
0.0
0π
1π
2π
3π
4π
P_2 (Ω_0 t), Δ =0
P_2 (Ω_0 t), Δ =Ω_0
P_2 (Ω_0 t), Δ =2Ω_0
time