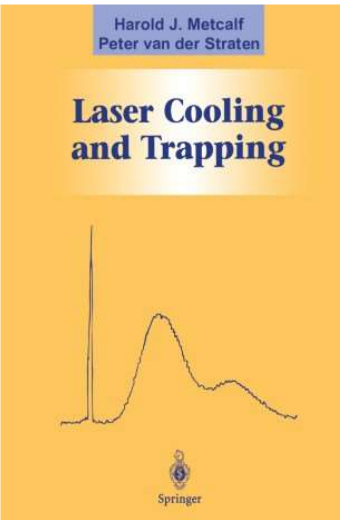
Optical Bloch Equations (OBE)

Mittwoch, 15. Dezember 2021 16:14



A good book!
Short chapter on DM & OBE

OBEs → time evolution of density matrix elements

$$i\hbar \frac{d}{dt}\hat{\rho} = [\hat{H}, \hat{\rho}]$$

1st. case: no decay, no damping

e.g. $\frac{d}{dt}\rho_{11} = \frac{d}{dt} c_1 c_1^* = \dot{c}_1 c_1^* + c_1 \dot{c}_1^*$

c_1 from last week (above)

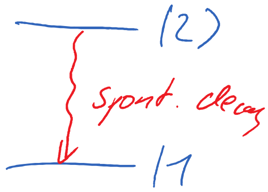
$$\rightarrow \dot{\rho}_{11} = i\frac{\Omega_0}{2}\left(e^{i\delta t}\rho_{21} - e^{-i\delta t}\rho_{12}\right)$$

RWA Rotating wave approx

$\tilde{\rho}_{12} = e^{i\delta t}\rho_{12}$ $\qquad$ $\tilde{\rho}_{11} = \rho_{11}$

$\tilde{\rho}_{21} = e^{i\delta t}\rho_{21}$ $\qquad$ $\tilde{\rho}_{22} = \rho_{22}$

$$\Rightarrow \boxed{\begin{aligned}
\dot\rho_{11} &= +\gamma\rho_{22} + i\frac{\Omega_0}{2}\left(\tilde\rho_{21}-\tilde\rho_{12}\right)\\
\dot\rho_{22} &= -\gamma\rho_{22} + i\frac{\Omega_0}{2}\left(\tilde\rho_{12}-\tilde\rho_{21}\right)\\
\dot{\tilde\rho}_{12} &= -\left(\frac{\gamma}{2}+i\delta\right)\tilde\rho_{12} + i\frac{\Omega_0}{2}\left(\rho_{22}-\rho_{11}\right) \longleftarrow w\\
\dot{\tilde\rho}_{21} &= -\left(\frac{\gamma}{2}-i\delta\right)\tilde\rho_{21} + i\frac{\Omega_0}{2}\left(\rho_{11}-\rho_{22}\right)
\end{aligned}}$$



phenomenologically introduce spontaneous decay

- population is transferred from upper to lower state with rate γ
- coherences are damped with rate $\frac{\gamma}{2}$

$$\gamma = \frac{1}{\tau} \quad \text{upper state lifetime}$$

We observe

- $\dot\rho_{11} = -\dot\rho_{22} \Rightarrow \rho_{11}+\rho_{22} = 1 = \text{const.}$
- $\rho_{12} = \rho_{21}^*$

We now introduce u, v, w

$$w = \rho_{22} - \rho_{11} \qquad \text{inversion}$$

$w = 1 \rightarrow$ upper
$0 \rightarrow$ 50:50
$-1 \rightarrow$ lower state

$$\dot{\tilde\rho}_{12} = -\left(\frac{\gamma}{2}-i\delta\right)\tilde\rho_{21} - \frac{iw\Omega_0}{2}$$

$$\dot w = -\gamma(w+1) - i\Omega_0\left(\tilde\rho_{21}-\tilde\rho_{12}\right)$$

dynamics

dynamics

(1) $t \ll \frac{1}{\gamma} \equiv \tau$

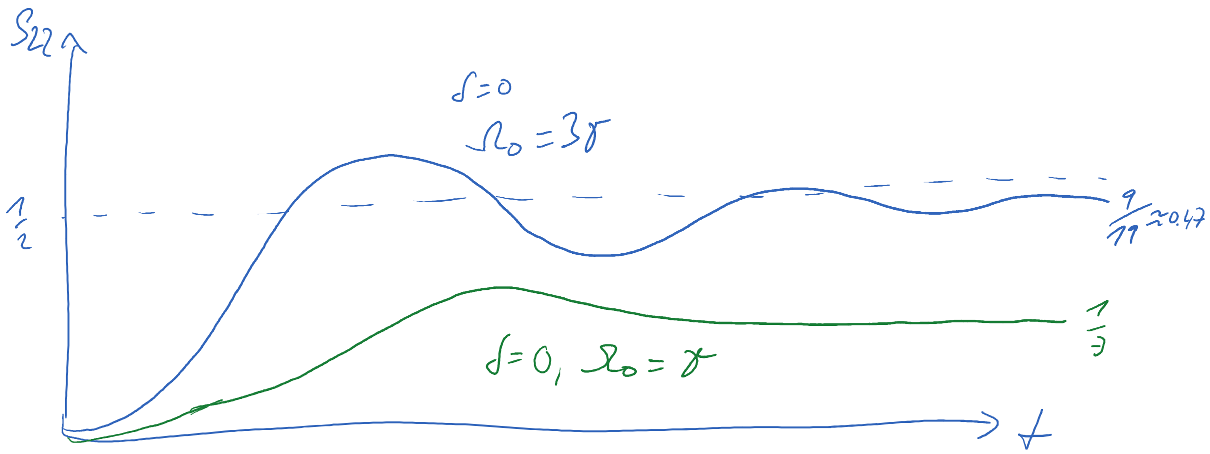
damping plays no role

Rabi oscillation if $\Omega_0 > \frac{1}{\gamma}$

(2) $t \gg \frac{1}{\gamma}$ steady state solution

coherences "die out"

populations constant in time



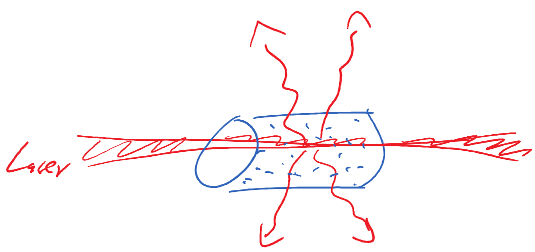
Steady-state of the OBEs

$\downarrow$

$$\dot{\tilde{\rho}}_{21} = 0 \quad , \quad \dot{w} = 0$$

L.H.S. of OBEs $= 0$

$$\boxed{\begin{aligned} w &= -\frac{1}{1+s} \\ \tilde{\rho}_{21} &= \frac{i\Omega_0}{2(\frac{\gamma}{2} - i\delta)(1+s)} \end{aligned}}$$



$s =$ saturation parameter

$$s = \frac{\Omega_0^2/2}{\delta^2 + \gamma^2/4} = \frac{s_0}{1 + 4\frac{\delta^2}{\gamma^2}}$$

s_0 = resonant saturation parameter

$\hookrightarrow \delta = 0$

$$s_0 = \frac{2\,\Omega_0^2}{\gamma^2}$$

limiting cases

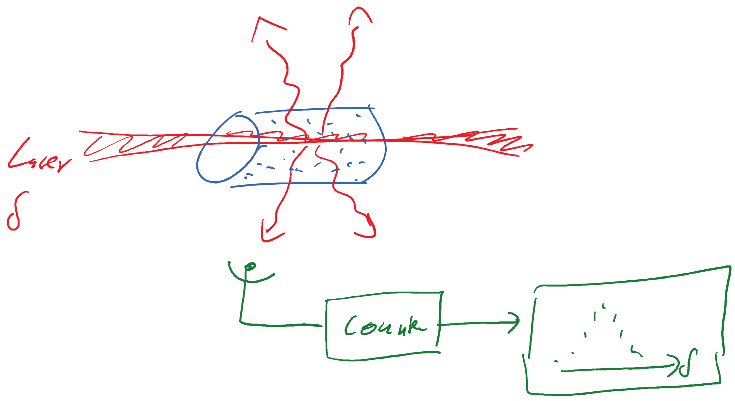
- low light intensity $\quad s \ll 1$

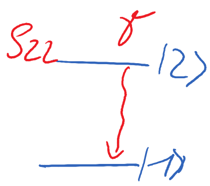
  $w = -1 \quad \rightarrow$ all atoms in g.s.

- $s \gg 1$:

  $w = 0 \quad \rightarrow$ 50:50 population

Photon scattering rate





$\Gamma_{ph} = \gamma \cdot \rho_{22}$ $\quad$ rate of excited state decay × probab. to be in the excited state

$$= \gamma \cdot \frac{1}{2}(1+w)$$

$$= \frac{s}{2(1+s)}\,\gamma$$

$$= \frac{\quad}{2(1+S)}\,\gamma$$

$$= \frac{S_0/2}{1+S_0+4\delta^2/\gamma^2}\cdot\gamma$$

$$= \frac{\gamma}{2}\left(\frac{1}{1+1/S_0}\right)\left(\frac{1}{1+4\delta^2/\gamma'^2}\right)$$

detuning

$$\gamma' = \gamma\sqrt{1+S_0}$$

FWHM of power-broadened line