Ex. 3

μp

w.f. overlap

$$P = 4\left(\frac{Z}{a_0}\right)^3 \int_0^{R_N} dr\, r^2\, e^{-2Zr/a_0}$$

$$= 1 - e^{-2ZR_N/a_0}\left(1 + \frac{2R_N Z}{a_0} + \frac{2R_N^2 Z^2}{a_0^2}\right)$$

μp: $m_\mu \approx 207\, m_e$

$R_N = R_p = 0.84\,\text{fm}$ $\qquad P(\mu p) = 4.6 \cdot 10^{-8}$

H $\qquad P(H) = 5.2 \cdot 10^{-15}$

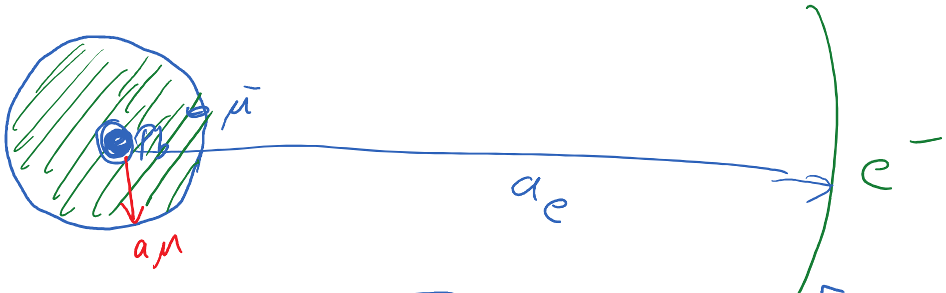
$\text{ratio} = 8.9 \cdot 10^{6}$

$= \left(\frac{m_\mu}{m_e}\right)^3$

$$a_B(\mu p) = \frac{m_e}{m_\mu} \cdot a_0$$

$\uparrow$

$m_{red} \approx 186$



~~p = 0.047~~

$$a_B(Z) = \frac{a_0}{Z} = \frac{53\,\text{pm}}{Z = 82} \in$$

$\leftarrow 81$

$$a_B(\mu Z) = \frac{1}{200 \cdot Z} \Leftarrow \frac{53\,pm}{200 \cdot 82}$$

$$\int_0^{R_p} \longmapsto \int_0^{a_B(\mu Z)}$$

$$P = 1 - e^{\frac{-2 \cdot 81 \cdot a_\mu}{a_0}} \left(1 + \frac{2 a_\mu}{a_e} + \frac{2 a_\mu^2}{a_e^2}\right)$$

$a_0 / Z = 81$

$$P = 1.6 \cdot 10^{-7}$$

$$C^{LM}_{1 m_1 1 m_2} = (-1)^L \, C^{LM}_{1 m_2 1 m_1}$$

$l_1 = 1 = l_2$

$L = 0, 2$

S D

$\underline{\underline{L = 1}}$

$M_j = \underline{\underline{m + \frac{1}{2}}}$

sum over all m, m_s

$j = l + s$

$m_s = +\frac{1}{2}$ up $M_j = m + \frac{1}{2}$

$$\Psi_{n,l,j=l+1/2,m_j} = \sqrt{\frac{l+m+1}{2l+1}} R_{nl} Y_{lm} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Leftarrow$$

$\ldots_{n,l,j\ldots,m_j} \quad \frac{\ldots}{2\ell+1} \, R_{n\ell} Y_{\ell m}\begin{pmatrix} \ldots \\ 0 \end{pmatrix}$

$$m_j = m' - \frac{1}{2} \qquad + \sqrt{\frac{\ell - m' + 1}{2\ell+1}} \, R_{n\ell} Y_{\ell m'} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$m' = m+1$$

b) $\vec{L} \cdot \vec{S} = L_x S_x + L_y S_y + L_z S_z$

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\vec{L} \cdot \vec{S} = \frac{\hbar}{2}\left[\begin{pmatrix} 0 & L_x \\ L_x & 0 \end{pmatrix} + \begin{pmatrix} 0 & -iL_y \\ iL_y & 0 \end{pmatrix} + \begin{pmatrix} L_z & 0 \\ 0 & -L_z \end{pmatrix}\right]$$

$$= \frac{\hbar}{2}\begin{pmatrix} L_z & \overbrace{L_x - iL_y}^{L_-} \\ \underbrace{L_x + iL_y}_{L_+} & -L_z \end{pmatrix} \qquad L_z = \underline{m \cdot \hbar}$$

$$\vec{L} \cdot \vec{S} \, \psi_{n,\ell,j=\ell+\frac{1}{2},m_j} =$$

$$= \frac{\hbar^2 m}{2} \sqrt{\frac{\ell+m+1}{2\ell+1}} \, R_{n\ell} Y_{\ell m} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$+ \frac{\hbar^2}{\ldots} \sqrt{\frac{\ell+m+1}{\ldots}} \sqrt{\ell(\ell+1) - m(m+1)} \, R_{n\ell} Y_{\ell, m+1} \begin{pmatrix} 0 \\ \ldots \end{pmatrix}$$

$$+\frac{\hbar^2}{2}\sqrt{\frac{l+m+1}{2l+1}}\sqrt{l(l+1)-m(m+1)}\,R_{nl}Y_{l,m+1}\begin{pmatrix}0\\1\end{pmatrix}$$

$$+\frac{\hbar^2}{2}\sqrt{\frac{l-m}{2l+1}}\sqrt{l(l+1)-m(m+1)}\,R_{nl}Y_{lm}\begin{pmatrix}1\\0\end{pmatrix}$$

$$-\frac{\hbar^2(m+1)}{2}\sqrt{\frac{l-m}{2l+1}}\,R_{nl}Y_{l,m+1}\begin{pmatrix}0\\1\end{pmatrix}$$

$$= \dots$$

$$= \frac{\hbar^2}{2}l\,\psi_{n,l,j=l+\frac{1}{2},m_j}$$

other one

$$-\frac{\hbar^2}{2}(l+1)\,\psi_{n,l,j=l-\frac{1}{2},m_j}$$

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c) compare $\quad \vec{L}\cdot\vec{S} = \frac{1}{2}\left(J^2-L^2-S^2\right) \leftarrow$

$$\hat{J}^2 \to \hbar^2 j(j+1)$$

$$\hat{S}^2 \to \hbar^2 s(s+1)$$

$$\hat{L}^2 \to \hbar^2 l(l+1)$$

$$\vec{L}\cdot\vec{S}\,\psi_{n,l,j,m} = \frac{\hbar^2}{2}\left(j(j+1)-l(l+1)-s(s+1)\right)$$

$j = l+s$ for $s=\frac{1}{2}$ $\quad \to \frac{\hbar^2}{2}l$

$$\frac{\hbar^2}{2}\left((l+\tfrac{1}{2})(l+\tfrac{3}{2})-l(l+1)-\tfrac{1}{2}(\tfrac{3}{2})\right)$$

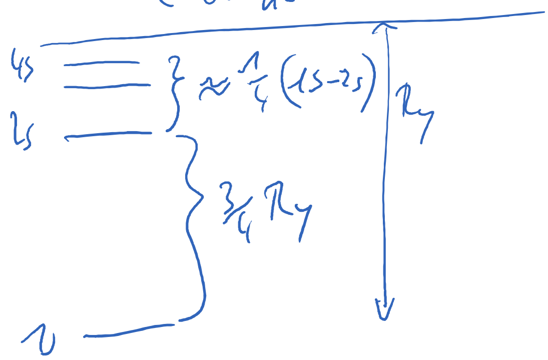
$$\frac{\hbar^2}{2}\left(\left(\ell+\tfrac{1}{2}\right)\left(\ell+\tfrac{3}{2}\right)-\ell(\ell+1)-\tfrac{1}{2}\left(\tfrac{3}{2}\right)\right)$$

4. #2 2.b. rotated Stern-Gerlach

$$\Delta E = \left(E_{4s}-E_{2s}\right)-\frac{1}{4}\left(E_{2s}-E_{1s}\right) = h\cdot 4797\,\mathrm{MHz}$$

$$= 1.98\cdot 10^{-5}\,\mathrm{eV}$$

$E \propto \frac{1}{n^2}$



s states $\ell = 0$

$j = s = \frac{1}{2}$

$$E_{1s} = \underbrace{E_{1j}}_{\text{DE}} + \underbrace{h\cdot L_{1s}}_{\text{Lamb shift}}$$

$$E_{2s} = E_{2j} + h\,L_{2s}$$

$$E_{4s} = E_{4j} + h\,L_{4s}$$

$$\text{DE:}\quad E_{nj} = -E_n\,\frac{Z^2\alpha^2}{n}\left(\frac{3}{4n}-\frac{1}{j+\frac{1}{2}}\right)$$

$$\hookrightarrow\quad E_{1j} = \underline{-1.81\cdot 10^{-4}\,\mathrm{eV}}$$

$$2j = -5.66\cdot 10^{-5}\,\mathrm{eV}$$

$$4j = -9.2 \cdot 10^{-6}\,eV$$

$$E_{4s} = -8.65 \cdot 10^{-6}\,eV \leftarrow$$

$$E_{2s} = -5.22 \cdot 10^{-6}\,eV \leftarrow$$

$$E_{1s} = 4 \cdot \Delta E - 4E_{4s} + 5E_{2s} = \underline{-1.47 \cdot 10^{-4}\,eV}$$

$$\nu_{1s} = \frac{-1.41 + 1.81}{h} \cdot 10^{-4}\,eV$$

$$= 8.2\ GHz$$