Ex5a

Exercise 8

08.12.2021

1 Resonant optical cavities

Hint: Please consider reading "Joseph T. Verdeyen - Laser Electronics (3rd Edition) - Prentice Hall (1995).pdf, chapter 6"

In the following Problem we will touch upon resonant optical cavities, which are fundamental building blocks when it comes to laser. Cavities are also used a lot within spectroscopy and atomic physics experiments. An optical cavity is constructed in the simplest way by two mirrors, the mirrors can take different shapes from flat to plano-concave.

One common type of cavity is the (plan , plano-concave, plano-convex). In this type of cavity as you see in the figure below you have one curved mirror and one flat mirror.

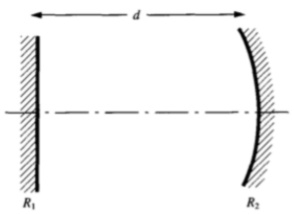

Figure 1: Plano concave cavity

with the distance between the two mirrors $d = \frac{R_2}{4}$, $\Gamma_1^2 = 0.99$, and $\Gamma_2^2 = 0.97$.

Here R_2 is the radius of curvature of the curved (plano-concave) mirror, a plano-concave mirror works in a similar way to a lens of a focal lens $= \frac{-R}{2}$, where R is the radius of curvature

Where $\Gamma_1^2 = 0.99$, and $\Gamma_2^2 = 0.97$. are the reflectivity of the mirrors respectively.

If you excite a cavity with a laser source of certain wavelength, a cavity can be in resonance which means the cavity can store the light for some time. This only happens if you satisfy certain condition.

(a) Define the first 4 transverse and longitudinal modes that occurs inside the cavity, just mention the name and draw a rough shape of them.

(b) What is the condition of a cavity with a length d to be in resonance with a laser source of a wavelength λ_0.

(c) How many modes can you fit into that cavity with $d = 75mm$ and $\lambda_0 = 1.5\mu m$.

(d) If the radius of curvature is 300 mm and the wavelength region of interest is $1.5\mu m$, compute the following quantities:

  (1) Free spectral range in MHz and in nm units
  (2) Cavity Q
  (3) Photon lifetime in nano sec
  (4) Finesse

2 Density Matrices

Density matrices are a convenient tool to express not only pure states (including superposition states), but also mixed states like e.g. thermal states. In the following we will consider an ensemble of two-level atoms with the two internal states $|g\rangle$ (ground state) and $|e\rangle$ (excited state).

(a) Assume that you have a an ensemble of atoms that are all in the state $|\Psi\rangle = \sqrt{3}/2|g\rangle - i1/2|e\rangle$. What is the density matrix of an atom in this ensemble in the basis $|g\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|e\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$?

(b) Assume now that you have created an ensemble by mixing 75% ground state atoms and 25% excited state atoms. If you pick one atom at random out of this ensemble, what is its density matrix?

(c) Assume that somebody gave you an ensemble of atoms. How could you experimentally distinguish between the above two situation?

(d) The entropy of a density matrix is defined as $S = -k_B \operatorname{Tr}(\hat{\rho}\log(\hat{\rho}))$. Calculate the entropy of the above two density matrices!

(e) How would the density matrix of a thermal state at $T = 0$ and $T = \infty$ look and what would be its entropy? A thermal density matrix is defined as $\hat{\rho} =$

$\frac{1}{Z}\sum_i e^{-\frac{E_i}{k_B T}}|i\rangle\langle i|$, where Z is the normalization constant which ensures that the sum of the diagonal elements is 1.