Bloch vector

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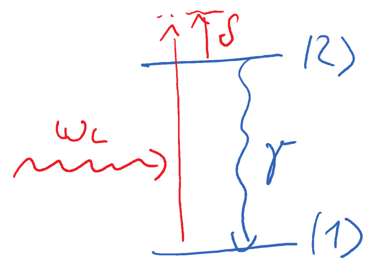
$$\boxed{\begin{aligned}
\dot{\rho}_{11} &= +\gamma \rho_{22} + i\frac{\Omega_0}{2}\left(\tilde{\rho}_{21} - \tilde{\rho}_{12}\right)\\
\dot{\rho}_{22} &= -\gamma \rho_{22} + i\frac{\Omega_0}{2}\left(\tilde{\rho}_{12} - \tilde{\rho}_{21}\right)\\
\dot{\tilde{\rho}}_{12} &= -\left(\frac{\gamma}{2} + i\delta\right)\tilde{\rho}_{12} + i\frac{\Omega_0}{2}\left(\rho_{22} - \rho_{11}\right)\\
\dot{\tilde{\rho}}_{21} &= -\left(\frac{\gamma}{2} - i\delta\right)\tilde{\rho}_{21} + i\frac{\Omega_0}{2}\left(\rho_{11} - \rho_{22}\right)
\end{aligned}}$$



u, v, w components of Bloch vector

$u = \tilde{\rho}_{12} + \tilde{\rho}_{21} = 2\,\mathrm{Re}(\tilde{\rho}_{12})$ dispersive

$v = i(\tilde{\rho}_{21} - \tilde{\rho}_{12}) = 2\,\mathrm{Im}(\tilde{\rho}_{12})$ absorptive

component of the Bloch vector

$w = \rho_{22} - \rho_{11}$ inversion

$\Rightarrow$ OBEs become

$$\begin{aligned}
\dot{u} &= \delta v - \frac{\gamma}{2} u\\
\dot{v} &= -\delta u + \Omega_0 w - \frac{\gamma}{2} v\\
\dot{w} &= -\Omega_0 v - \gamma (w+1)
\end{aligned}$$

$\hat{=}$

$$\frac{d}{dt}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \underbrace{-\begin{pmatrix} \Omega_0 \\ 0 \\ \delta \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix}} - \gamma \begin{pmatrix} u/2 \\ v/2 \\ w+1 \end{pmatrix}$$

Precession of the Bloch vector around axis given by $\begin{pmatrix} -\Omega_0 \\ 0 \\ -\delta \end{pmatrix}$ $\leftarrow$ rotation axis

with angular frequency $\Omega = \sqrt{\Omega_0^2 + \delta^2}$ (generalized Rabi frequ.)

without damping ($\gamma = 0$): $u^2 + v^2 + w^2 = 1$

$\rightarrow$ Bloch vector is moving on a <u>unit sphere</u>

<u>Examples</u>

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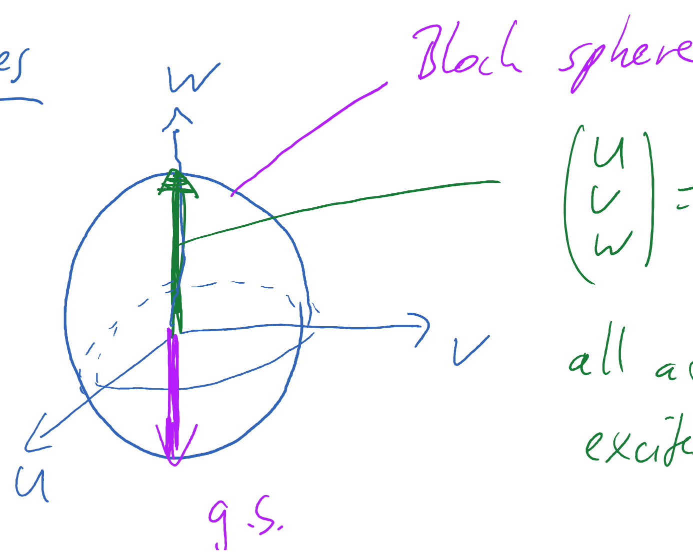
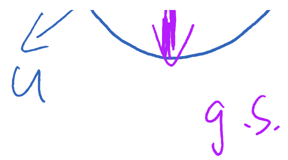


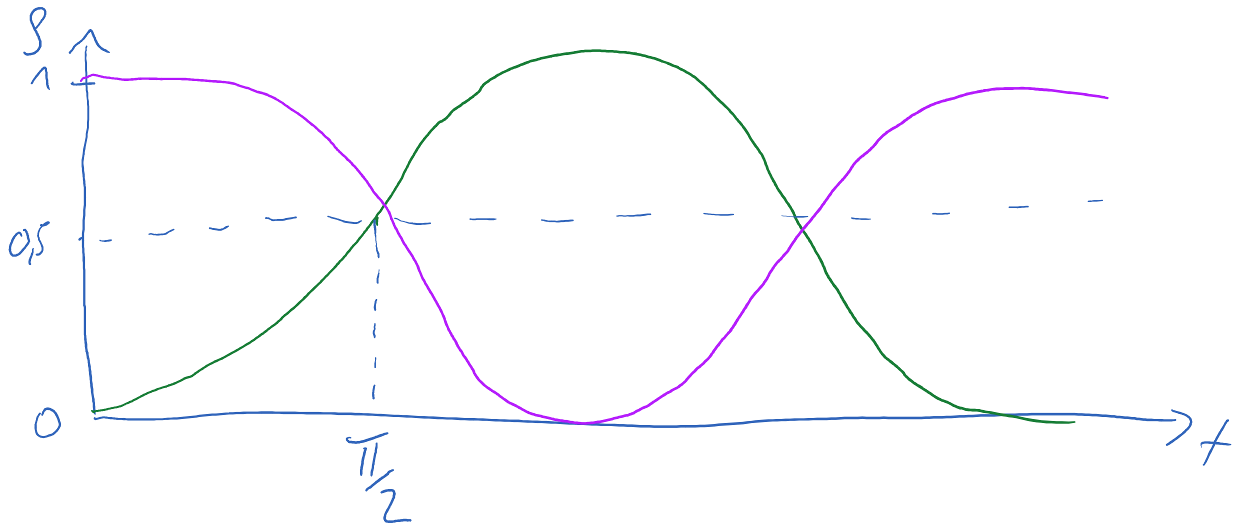
$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

all atoms in the excited state



excited state

② atoms in g.s., $\frac{\pi}{2}$ pulse on resonance $(\delta = 0)$



coherent superposition of g.s. & excited state

$$(u, v, w) = (0, -1, 0)$$

$$\tilde{c}_1 = \cos\left(\frac{1}{2}\Omega_0 t\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\tilde{c}_2 = i\sin\left(\frac{1}{2}\Omega_0 t\right) = i\sin\frac{\pi}{4} = i\frac{1}{\sqrt{2}}$$

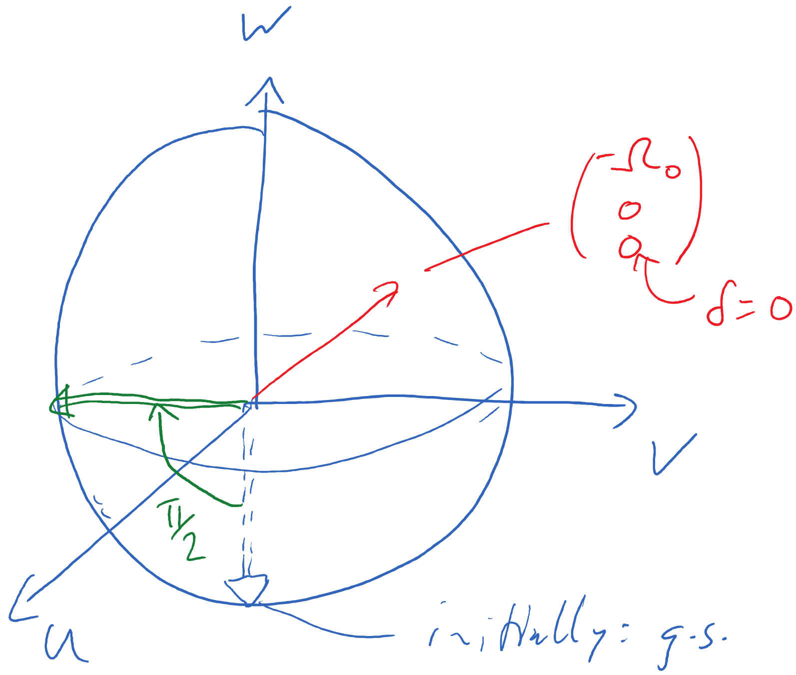
$$\rightarrow \rho_{11} = \tilde{c}_1\tilde{c}_1^* = \frac{1}{2} \qquad \rho_{22} = \tilde{c}_2\tilde{c}_2^* = \frac{1}{2} \rightarrow \underline{\underline{w = 0}}$$

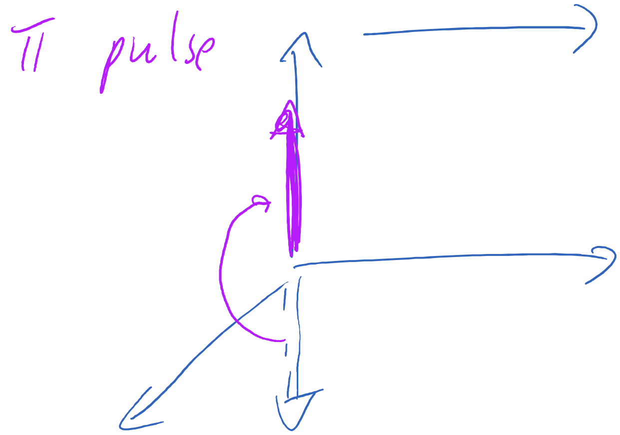
$$\tilde{\rho}_{12} = \tilde{c}_1\tilde{c}_2^* = -\frac{i}{2}$$

$$\tilde{\rho}_{21} = \tilde{c}_2\tilde{c}_1^* = \frac{i}{2}$$

$$u = \tilde{\rho}_{12} + \tilde{\rho}_{21} = -\frac{i}{2} + \frac{i}{2} = \underline{\underline{u = 0}}$$

$$v = i\left(\tilde{\rho}_{21} - \tilde{\rho}_{12}\right) \qquad \underline{\underline{v = -1}}$$





$$(u, v, w) = (0, 0, 1)$$

a) resonant excitation ($\delta = 0$)

starting from g.s. $(0, 0, -1)$

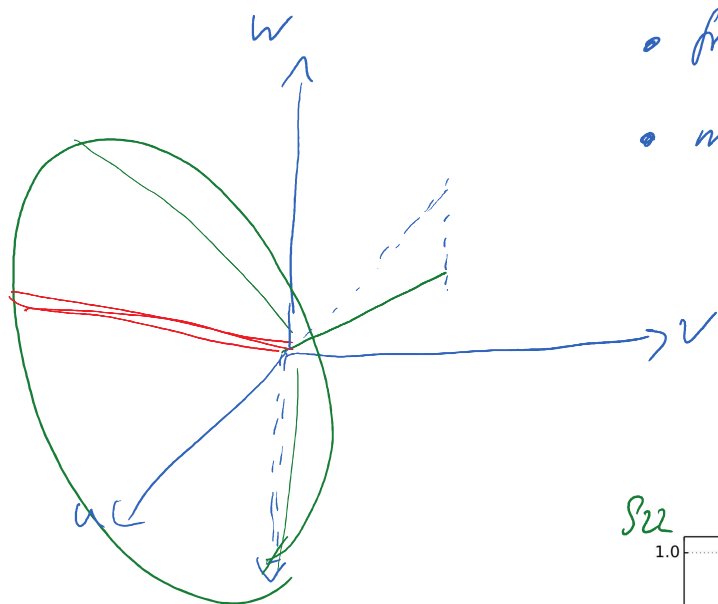
the Bloch vector moves along circle through poles of Bloch sphere

$\Rightarrow$ Rabi oscillations

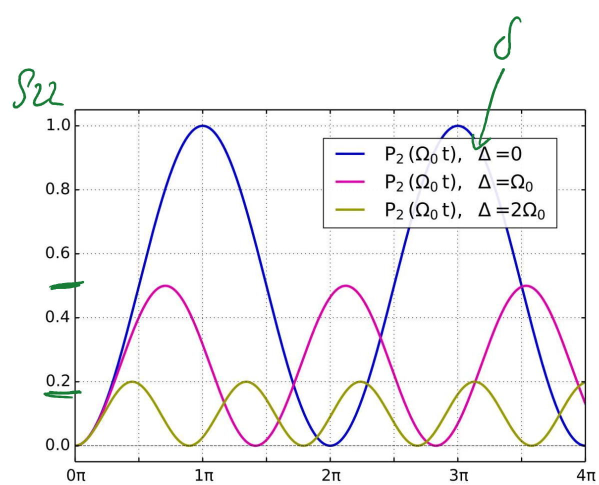
b) non-resonant excitation ($\delta \neq 0$)

rotation axis $\neq$ $-u$ axis but

$(-\Omega_0, 0, \underline{-\delta})$



- frequency $\Omega = \sqrt{\Omega_0^2 + \delta^2}$
- modulation not fully modulated
  $\rightarrow$ Bloch vector does not reach the north pole



Note: For $\delta \neq 0$ the Bloch vector is still rotating with angular frequency $\underline{\underline{\delta}}$ around $\underline{w}$ axis

"relic" from rotating wave approximation