Ex5a

Exercise 10

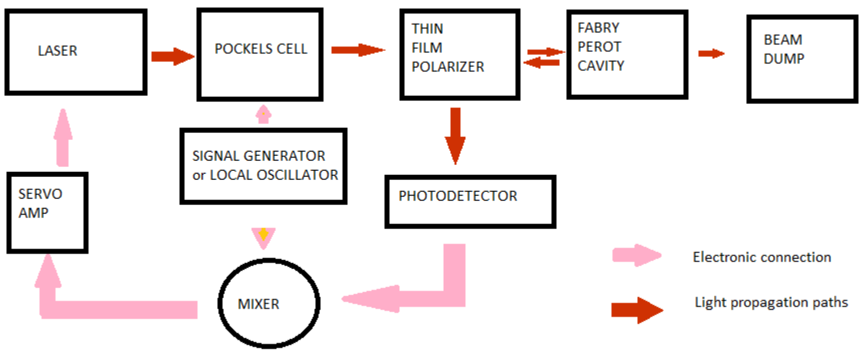
10.1 Rabi frequency

An atomic ensemble of hydrogen atoms is adressed by a laser tuned to resonance with the 1s - 2p transition at 121 nm. The dipole moment of this transition is $\mu_{12} = 6.32 \times 10^{-30}$ Cm.

a) Calculate the Rabi oscillation period for $I = 10\,\mathrm{kWm^{-2}}$. Assume that the light is polarized in z-direction. Compare your value with *typical* lifetimes for E1 (electric dipole-allowed) transitions.

b) Calculate the optical intensity required to make the Rabi oscillation period equal to the radiative lifetime of the 2p level, namely 1.6 ns.

10.2 Pound Drever Hall Locking



1. In practise, laser frequency deviates from the intended frequency when we run it continously. Thus it is necessary to stabilize the laser frequency. This can be done by locking the laser to a highly stable cavity or an atomic transition. Among the methods to do this are Pound Drever Hall locking (PDH locking), side by fringe locking and Hansch-Couillaud locking. We shall focus on PDH locking a laser to a cavity. We omit certain technicalities (such as those related to Servo Amp, laser controller, etc...) to focus more on optical aspects.

Consider a laser that we want to operate at 785nm. To make sure that it maintains its wavelength at 785nm, we need to measure its wavelength (or frequency) and report any deviations from the desired wavelength back to the laser controller, which will account for this deviation and make sure laser provides 785nm. Laser light can be fed to a cavity understand its frequency profile.

a) Determine the reflectance coefficient of a simple two-mirror optical cavity in terms of reflectance amplitude, free spectral range of cavity and angular frequency of input light

b) Plot the transmission and reflectance profile (magnitude and phase) of a simple two-mirror optical cavity

We can see that by measuring the reflected light and finding how it varies when we dither the frequency, one can determine whether the laser is at resonance to a cavity's longitudinal mode or not. More importantly, we can determine by how much the laser controller needs to change the frequency to make the laser gives out the desired frequency. One can measure the reflected signal from the cavity using a thin-film polarizer and a photodiode.

In practise, instead of dithering the frequency , phase modulation is much easier to implement with a Pockels cell. (A Pockels Cell can be thought of as a voltage controlled waveplate. A local oscillator is simply a signal generator that provides an AC voltage across the Pockel's cell. Thus the electric field of the laser (of angular frequency ω) comes out with a time dependent phase of $\omega t + \beta \sin(\Omega t)$, where β is called the modulation depth and Ω is called the modulation frequency). After the Pockel's cell, effectively the laser consists of several sidebands each spaced from the input laser frequency by integral values of modulation frequency. (The purpose of dithering the frequency of laser is to produce sidebands with a definite phase relation with the incident laser, which can be interfered with the reflected beam to measure the phase of the reflected beam. This is essentially acheived by phase modulation with a Pockels cell). Reflected photodetector signal and the local oscillator's signal are mixed electronically using a mixer. Electronically, a mixer retains the product of the 2 input signals. A low-pass filter chooses the low frequency part of mixer's output signal and feeds it to the Servo Amp (which is the laser controller).This signal is called the PDH error signal.

Please read (at least sections 1,2,3,4.A,4.B) of
An introduction to Pound–Drever–Hall laser frequency stabilization by Eric D. Black, Am. J. Phys., Vol. 69, No. 1, January 2001

(d) Derive eqn 3.3 in the above reference and explain how the error signal is isolated

2. Consider a two-mirror optical cavity consisting of 2 mirrors of reflectivity - 0.999 and 0.999 separated by a free-space of 1cm. Let 2 mW of a laser of wavelength 785nm and linewidth 1 MHz be directed into a Pockel's cell and subsequently to this cavity for PDH locking. For a phase modulation frequency of 520 MHz and modulation depth of 0.5,

a) find the frequency of first sideband and power of each sideband

b) find the magnitude of error signal when the laser frequency is dithered by a magnitude of half the cavity linewidth

c) plot the error signal vs laser frequency for modulation frequencies 52 MHz, 100MHz , 520 MHz and 1 GHz