Ex5a

Exercise 11

Atoms in a magnetic field

The Hamiltonian for an atom in a magnetic field along $\hat{z}$ may be written

$$H = ah\frac{\vec{I}\cdot\vec{J}}{\hbar^2} + \left(g_J\mu_0 m_J - g_I\mu_0 m_I\right)B_z$$

where a is the hyperfine interaction constant.

(a) Restrict attention to the case $J = 1/2$, but arbitrary I. Show that the energies of states are given by the Breit-Rabi formula

$$E_m^{\pm} = -\frac{ah}{4} - mg_I\mu_0 B_z \pm \frac{ahF^+}{2}\sqrt{1 + \frac{2mx}{F^+} + x^2}$$

x represents a dimensionless measure of the magnetic field strength relative to the hyperfine interaction strength.

The parameter x is given by $x = (g_I + g_J)\,\mu_0 B_z/ahF^+$, where $F^+ = I + 1/2$.m is the z component of the total angular momentum.
Recall that $\vec{I}\cdot\vec{J} = I_xJ_x + I_yJ_y + I_zJ_z$ It may also be helpful to recall that $I_\pm = I_x \pm iI_y$ are raising and lowering operators for the angular momentum operators.
Use any programming language to solve for the Hamiltonian after you write the matrix elements.

(b) For the case of $I = \frac{1}{2}$ of the hydrogen atom, make a clear sketch of energies vs the magnetic field B. Take advantage of the non-crossing rule (levels of the same m do not cross). Be sure to extend your figure to very high field $(1/x \ll g_I/g_J)$ (Paschen-Back effect). Label the lines with quantum numbers at low and high fields and indicate m.
Do this for the case of the hydrogen atom, where you will use the g factors of the electron and proton respectively. Be careful when substituting for g_I.

(c) For the hydrogen atom as above in the ground state, using the definition of x in part(a), is a magnetic field of 1 Tesla considered to be high field or low field region?

(d) one can also infer the effective magnetic moment μ_H of hyperfine state h, given by $\mu_H(h) = \frac{\partial E_h}{\partial B}$. show the effective magnetic moment (in units of the bohr magneton μ_B) as a function of the magnetic field B.

(e) Take a look back at the states you have drawn vs the magnetic field in part b, Which of these states are attracted to high magnetic field, and which are attracted to low magnetic field ? (also called high and low field seekers).
Which of these states we can use to trap Hydrogen in a magnetic field and why?.
Can you trap the states which are attracted to higher fields? why and why not?