Ex5a

Exercise 12

9.1 Zeeman Slower

The magnitude of the scattering force on an atom due laser cooling at a detuning δ from the atomic resonance can be written as

$$|\vec{F}_{\text{scatt}}| = \hbar k \frac{\gamma}{2} \frac{S_0}{1 + S_0 + \left(2\frac{\delta}{\gamma}\right)^2} = \hbar k \Gamma_{\text{scatt}}$$

where $k = \omega / c$ with the laser frequency ω, γ is the spontaneous decay rate of the excited state and $S_0 \propto I$ is the saturation parameter.

Consider a thermal beam of atoms (with a resonance frequency ω_0) moving along the z-axis through a magnetic field $B(z)$, where they interact with a counter-propagating laser beam. We define an 'unperturbed' laser detuning $\delta_0 := \omega - \omega_0$.

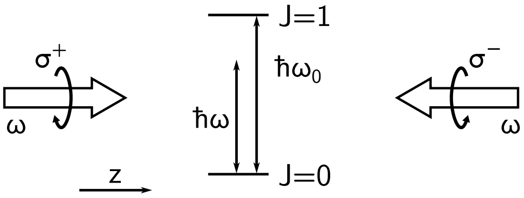
(a) Determine the 'effective' detuning $\delta(z)$ for the laser interacting with an atom at velocity $v(z)$, such that the resonance condition reads $\delta(z) = 0$. For the case that the laser is tuned to $\delta_0 = 0$, write down the resonance condition linking the magnetic field $B(z)$ to atoms at velocity $v(z)$.

(b) Determine the maximum deceleration an atom can experience due to laser cooling.

(c) Determine the appropiate magnetic field $B(z)$ for the Zeeman slower in case of adiabatic cooling, i.e. the deceleration is maximal for all coordinates z. Hint: $a(z) = v(z)\frac{\partial}{\partial z}v(z)$.

9.2 Scattering Force in Laser Cooling and Trapping

Consider a $J = 0 \leftrightarrow J' = 1$ transition in an atomic model system with resonance frequency ω_0. The atoms are interacting with a pair of red-detuned counterpropagating laser beams with polarizations σ^+ and σ^-. We restrict to one dimension (z-axis, see below), to reduce complexity. Further, we assume $J = 0 \leftrightarrow J' = 1$ to be a typical optical transition, a saturation parameter $S_0 \approx 1$ and a laser at $\delta_0 = \omega - \omega_0 = -2\pi \times 30\,\text{MHz}$.



In case of an absent magnetic field, an optical molasses is created.

(a) Using the scattering force F_{scatt} (see 9.1), show that the force on an atom with velocity v is given by $F_{\text{OM}} \approx -\beta v$ with a factor $\beta = 2k\frac{\partial}{\partial\omega}F_{\text{scatt}}(\omega - \omega_0)$. For this, assume a Doppler shift, which is small w.r.t the detuning δ_0.

(b) Using $\Gamma_{\text{scatt}} \ll \omega\frac{\partial}{\partial\omega}\Gamma_{\text{scatt}}$ (validate this approximation), determine the proportionality constant β and give an estimated numerical value.

(c) Make a plot of $F_{\text{OM}}(v)$ comparing the result with and without the approximation applied in part (a).

(d) Explain why this configuration is not capable of trapping atoms.

(e) What modification to the setup is need to be made in order to establish Sisyphus cooling? What's the advantage gained?

For realization of a one-dimensional magneto-optical trap (MOT), a linear magnetic field $B(z) = B_z z$ is applied. The Zeeman splitting of energy levels is assumed to be described by the low-field approximation.

(f) Similar to part (a), deduce an expression for the force F_{MOT} on an atom with velocity v at position z. Again, assume all shifts are small compared to the detuning δ_0. What kind of classical physical system is modelled here?

9.3 Magneto-optical Sodium Trap

The first magneto-optical trap was successfully demonstrated with sodium employing $3S_{1/2} \leftrightarrow 3P_{3/2}$ transitions in the D$_2$-line (near 589 nm) [1]. The trap was loaded with laser-evaporated pulses of atoms, which where cooled down by chirping a cooling laser.

Sodium is a hydrogen-like atom and can therefore be described as a one-electron system with nuclear spin $I = 3/2$. The corresponding selection rules for optical dipole transitions within JI-coupling include $\Delta F = 0, \pm 1$ (not $0 \leftrightarrow 0$), $\Delta m_F = 0, \pm 1$ (not $0 \leftrightarrow 0$ if $\Delta F = 0$) and $\Delta L = \pm 1$.

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[1] E. L. Raab *et al.*, Phys. Rev. Lett. **59**, 2632 (1987)

(a) Sketch the level diagram of the $3S_{1/2}$, $3P_{1/2}$ and the $3P_{3/2}$ state including the hyperfine structure ($A_{\mathrm{HFS}} > 0$ for all states).

(b) In case of the $3S_{1/2}(F = 2) \leftrightarrow 3P_{3/2}(F' = 3)$ hyperfine transition, make a sketch of the Zeeman splitting in the low-field regime and explain how a magneto-optical trap can be realized here.

(c) Explain, why an additional laser was nescessary to operate a magneto-optical trap at the $3S_{1/2}(F = 2) \leftrightarrow 3P_{3/2}(F' = 2)$ transition.