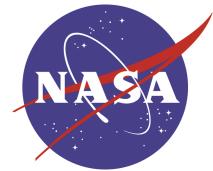


The Physics of Atomic Clocks

Eric Burt

Jet Propulsion Laboratory, California Institute of Technology



Joint European Frequency and Time Forum and IEEE International Frequency
Control Symposium

April 14, 2019

Tutorial Overview

- Atomic physics relevant to clocks
- Physics of key atomic clock techniques
- Key frequency shift effects in clocks
- Some fundamental physics tests using clocks

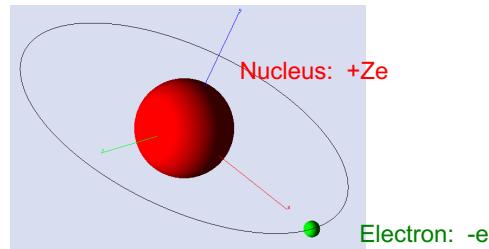
Atomic Clock Physics

Basic Microwave Atomic Clock Physics: Atomic Interactions

- Hyperfine transitions: 1-40 GHz
- Atomic interactions:

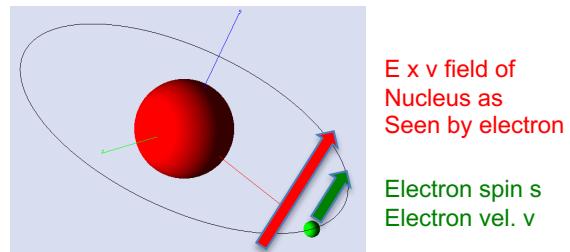
- **Coulomb** interaction

$$\Delta E = -\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r}$$

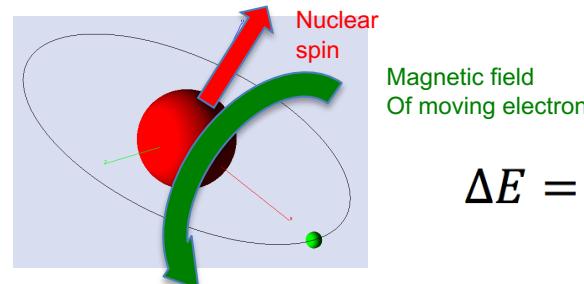


- **Fine structure**: electron spin interacts with nuclear electric field (optical)

$$\Delta E = -\bar{\mu}_S \cdot \bar{B} = \frac{2\mu_B \bar{s}}{c^2} \cdot (\bar{E} \times \bar{v})$$

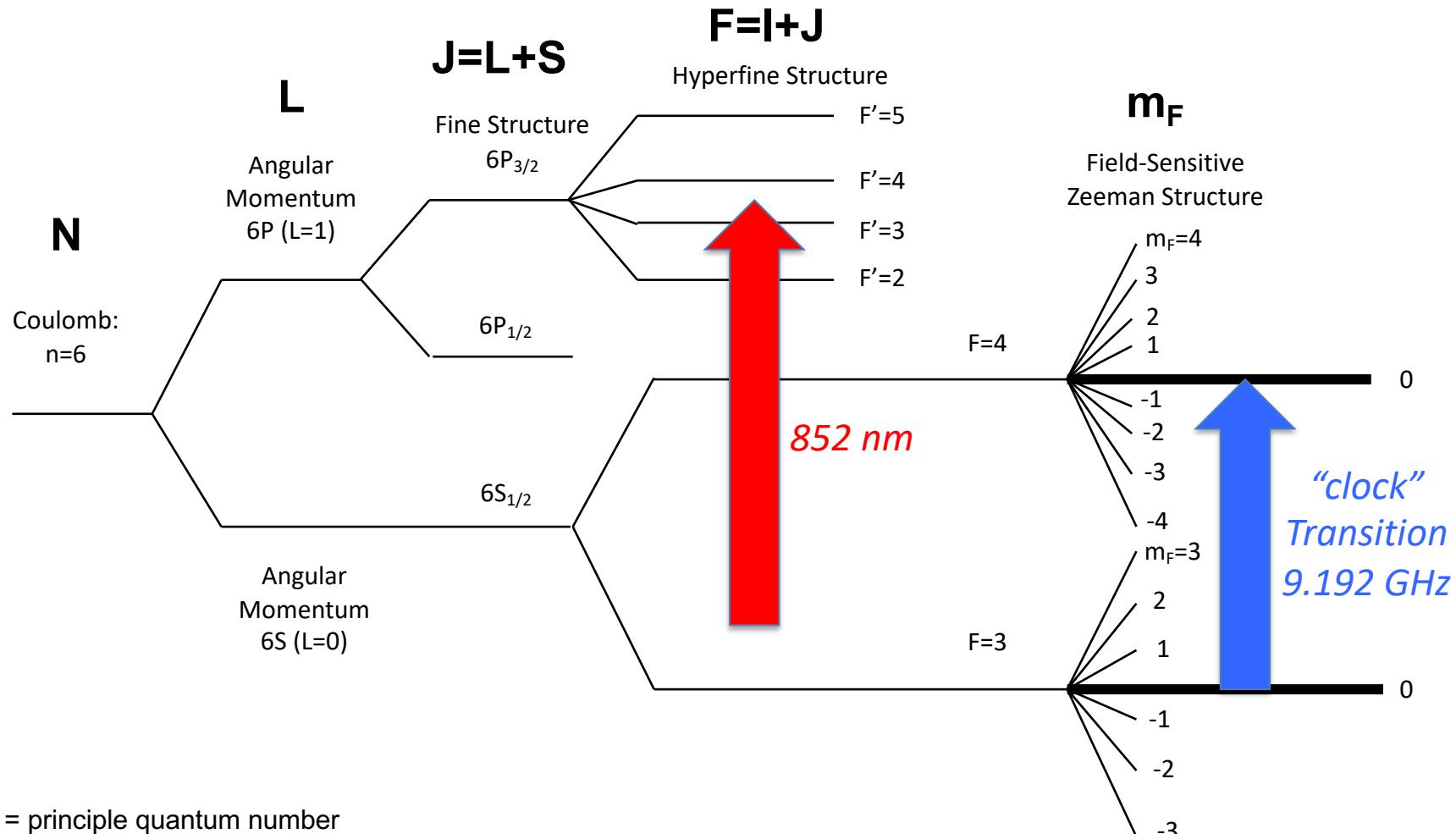


- **Hyperfine structure**: nuclear spin interacts with the magnetic field created by the moving electron (microwave)



$$\Delta E = -\bar{\mu}_N \cdot \bar{B}_{el}$$

Basic Microwave Atomic Clock Physics: Simplified Cesium Atomic Level Structure



N = principle quantum number

L = angular momentum quantum number

S = electron spin quantum number

I = nuclear spin quantum number

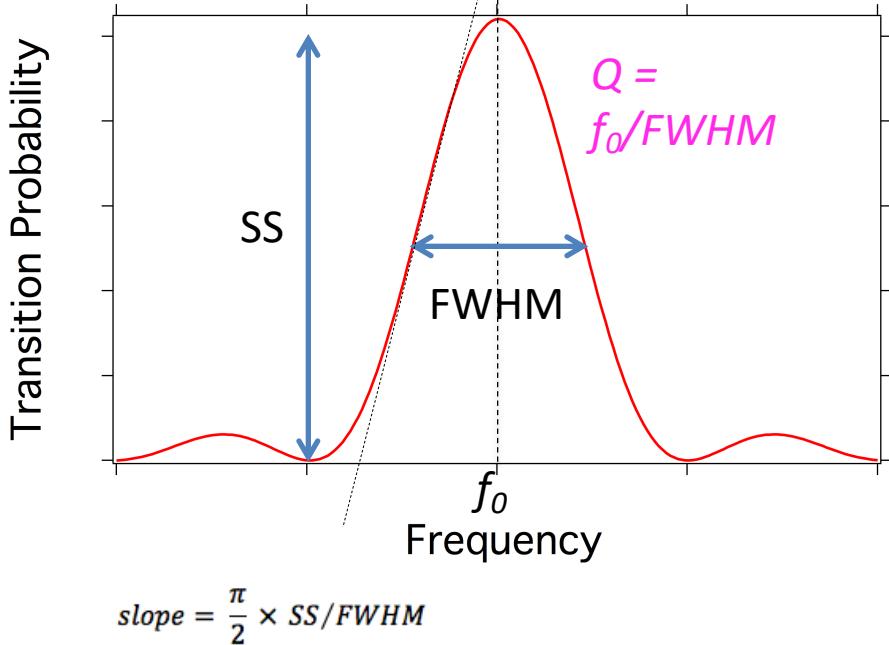
J = $L+S$: total electron angular momentum

F = $I+J$: total atomic angular momentum

Integral $F \Rightarrow$ existence of first-order field-insensitive $m_F=0 - m'_F=0$ transition

Clock transition line shapes: Clock stability and the Allan Deviation

Rabi interrogation:



$$\sigma(\tau) = \frac{1}{2} [\langle y(t + \tau) \rangle - \langle y(t) \rangle]^2)^{1/2}$$

$$\sigma(\tau = T_c) = \frac{\Delta f}{f_0} = \frac{\Delta SS}{f_0 \times \text{slope}} = \frac{1}{\pi SNR Q}$$

$$\text{slope} = \frac{\pi}{2} \times SS/FWHM$$

$$\Rightarrow \sigma(\tau) \rightarrow \frac{1}{\pi SNR Q} \sqrt{\frac{T_c}{\tau}} \text{ (White freq.)}$$

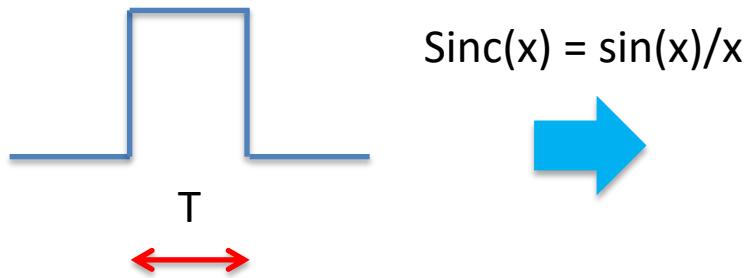
$$\sigma(1 \text{ sec}) \propto \frac{1}{SNR Q}$$

- **SNR** is at $SS/2$ and is limited by number of atoms
- f_0 is resonance frequency – choice of atom
- **FWHM** is linewidth – measurement time limited

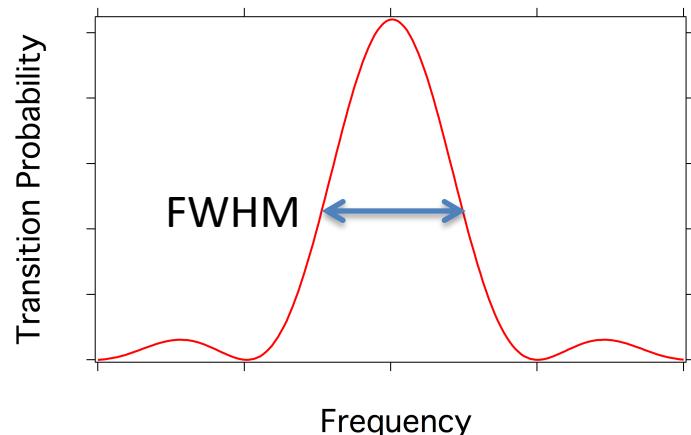
Example	Q	SNR	Adev (1s)
Rb	2e7	3000	2e-11
Cs	1e7	24000	5e-12
H-maser	2e9	5000	1e-13
Hg ⁺	5e12	20	2e-14

Ramsey vs. Rabi Interrogation

Rabi: microwave pulse on for T

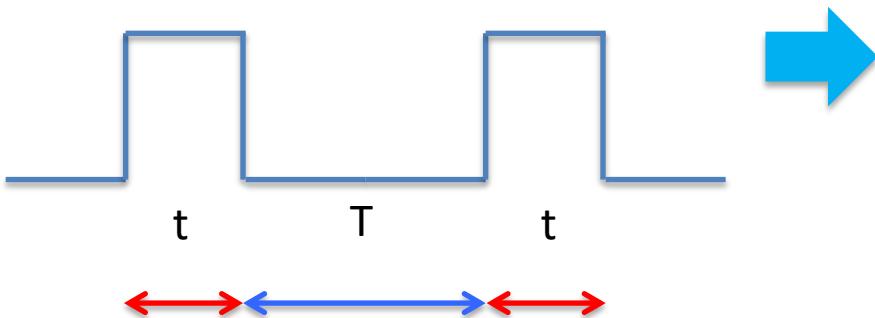


Rabi Pulse = 1 s

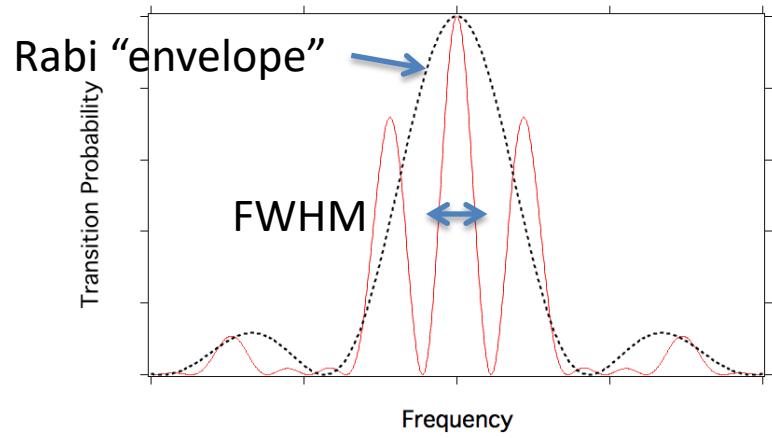


Ramsey: a time-domain interferometer:

two microwave pulses on for t separated by time T

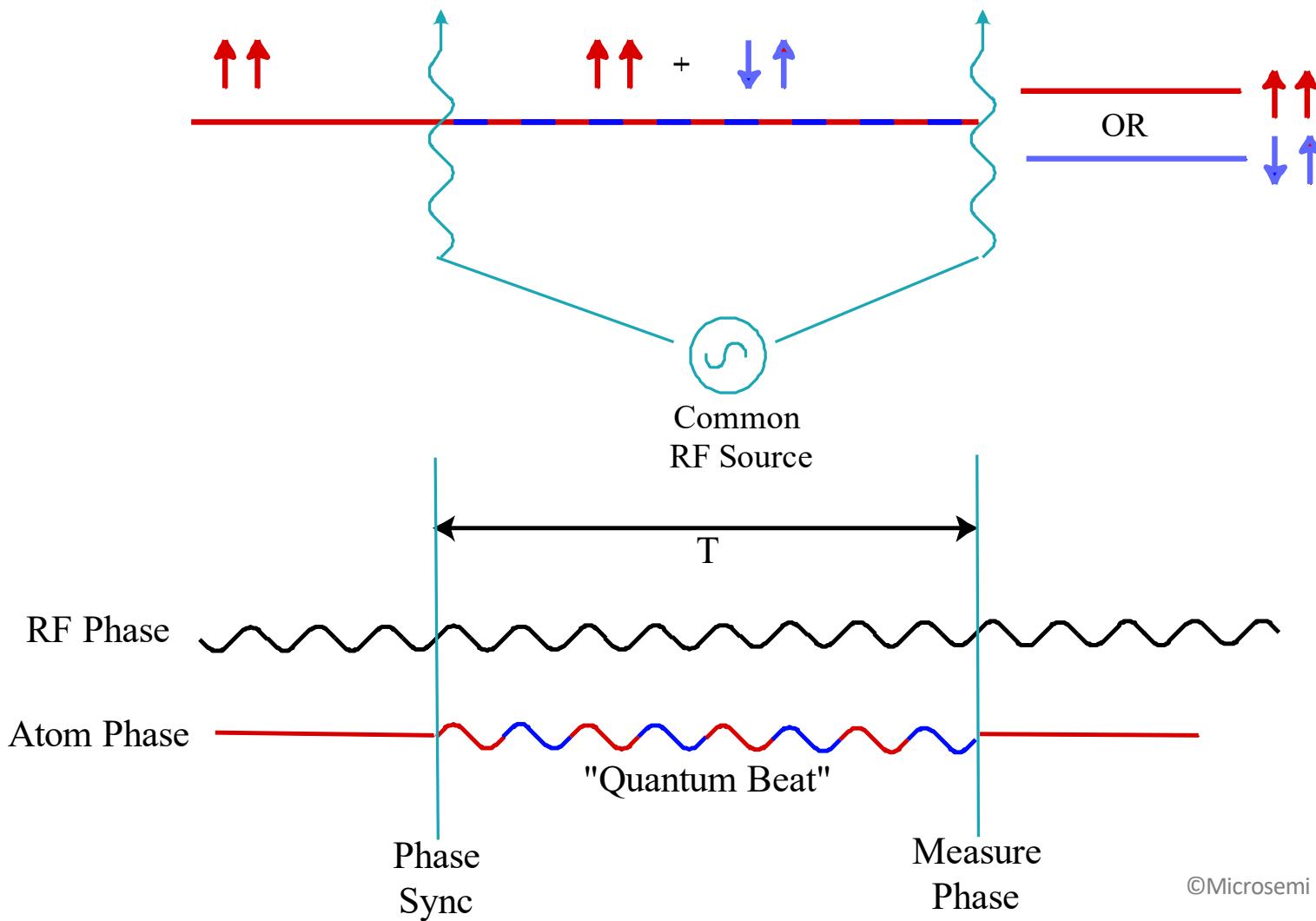


Ramsey = 1 s + 1 s + 1 s



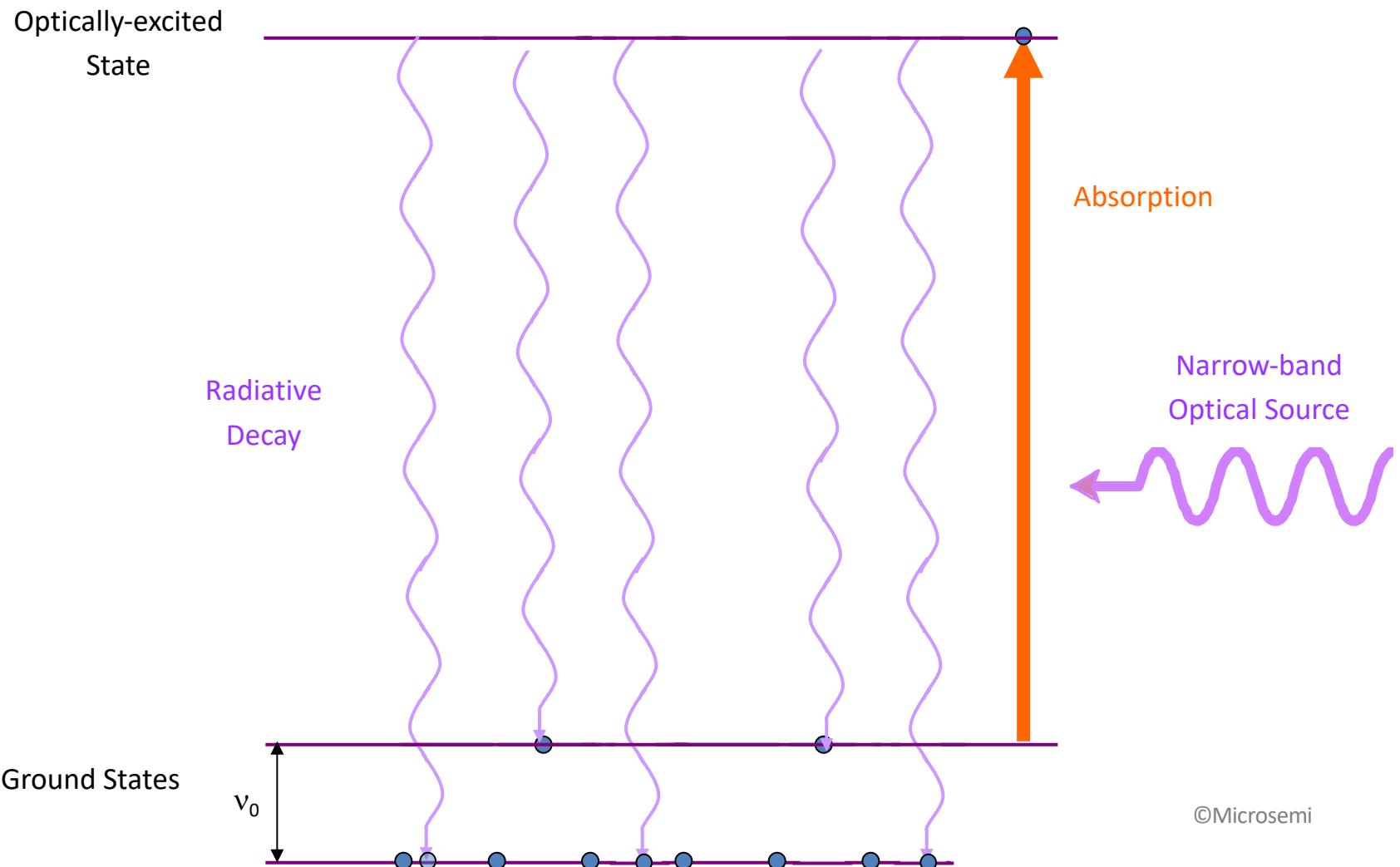
Basic Microwave Atomic Clock Physics

Ramsey Separated Oscillatory Fields



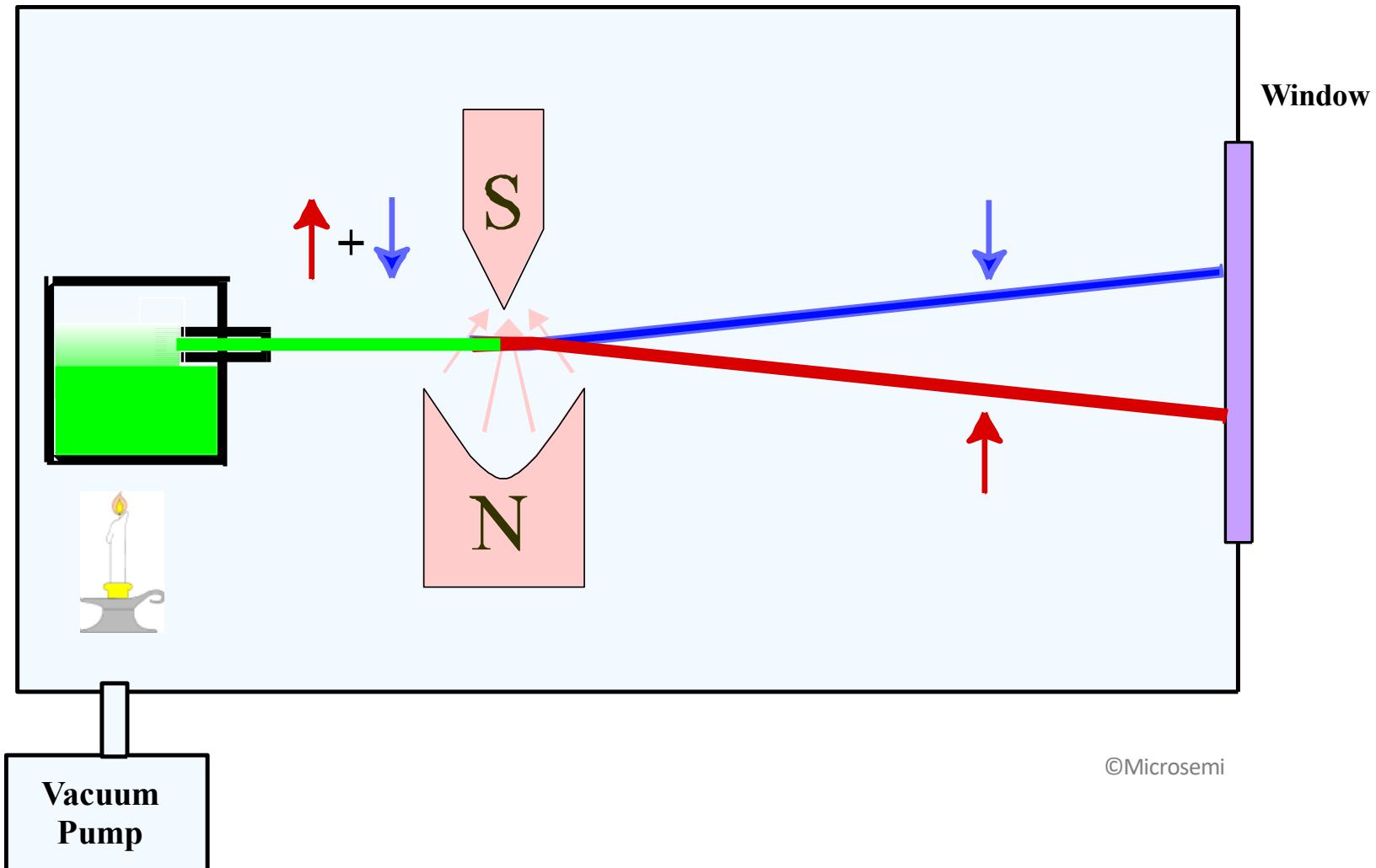
State selection:

Initialize into the right state using optical pumping



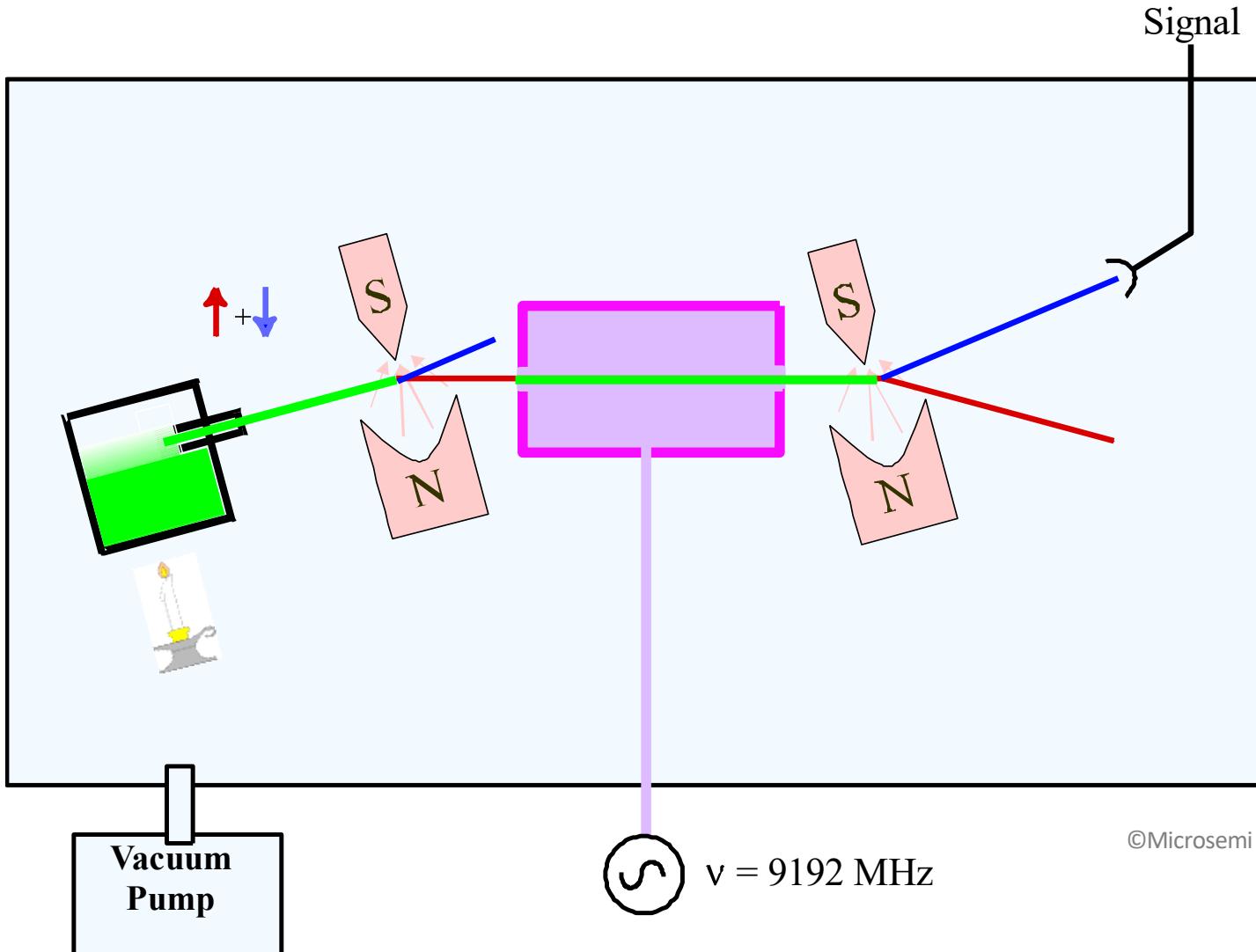
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Cesium Beam: The Stern Gerlach Effect (magnetic state selection)



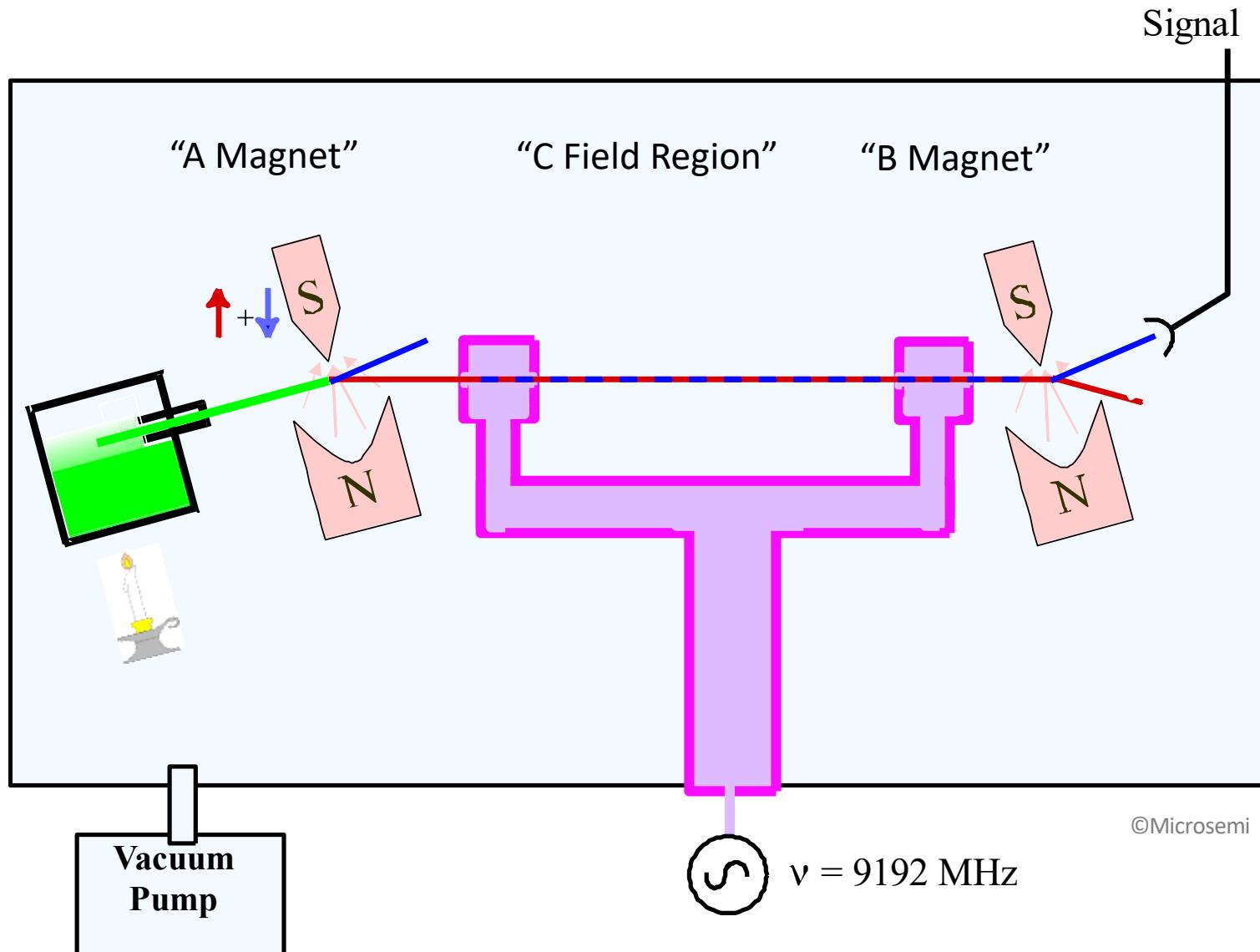
©Microsemi

Rabi Beam Magnetic Resonance



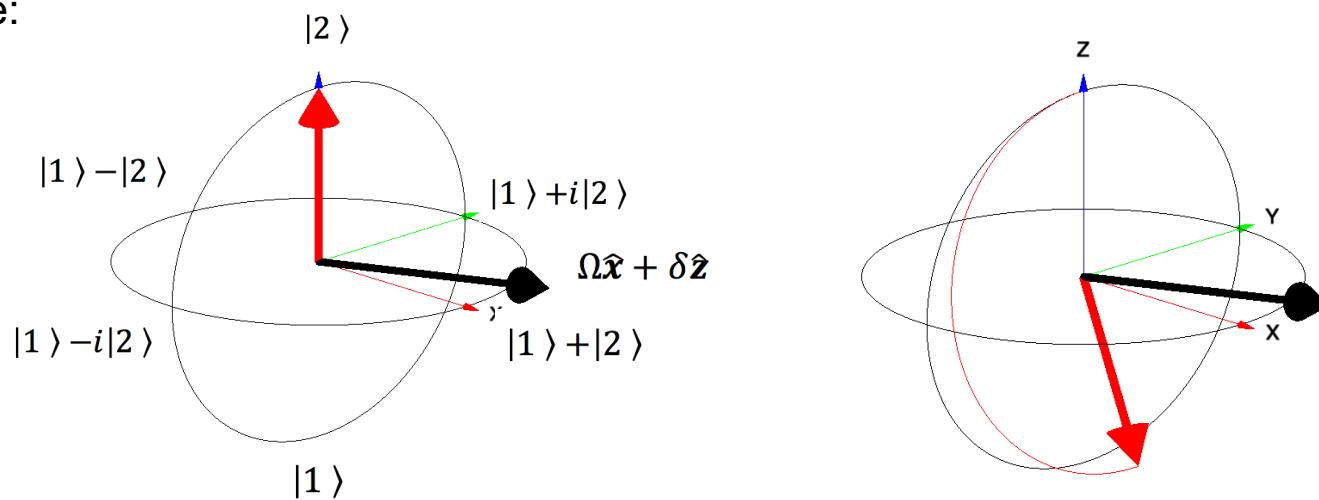
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Ramsey Magnetic Resonance: Separated Oscillatory Fields



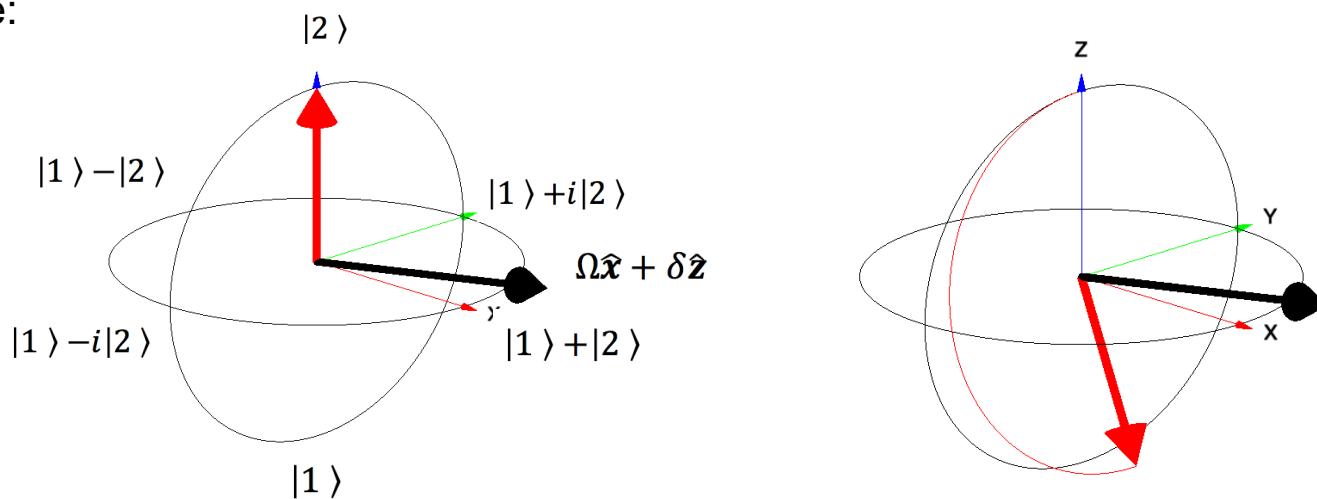
The Bloch Sphere: Representing the internal atomic state

Rabi 2 s pi-pulse:

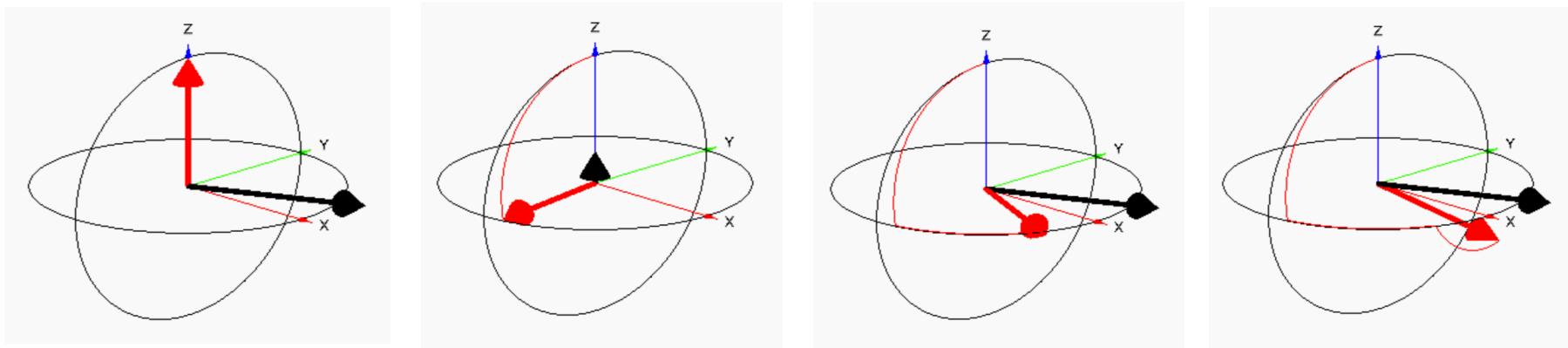


The Bloch Sphere: Representing the internal atomic state

Rabi 2 s pi-pulse:



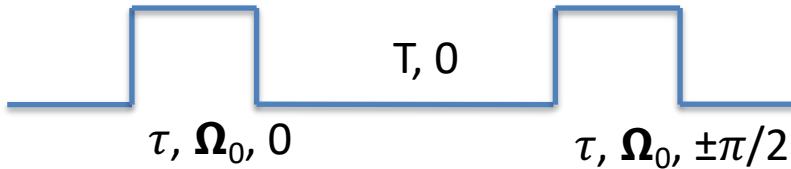
Ramsey 1 s pi/2-pulse, 4 s free, 1 s pi/2-pulse:



The Bloch Sphere: Extensions to Ramsey Interrogation

- elimination of the interrogation light shift

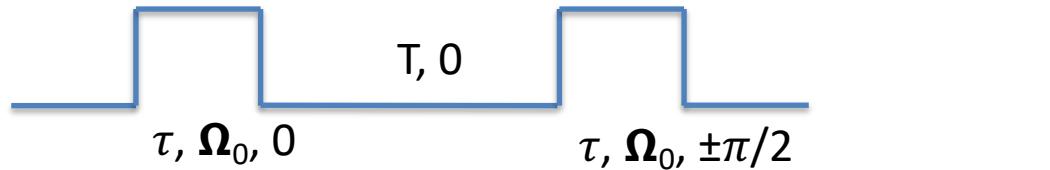
Ramsey (phase modulation):



The Bloch Sphere: Extensions to Ramsey Interrogation

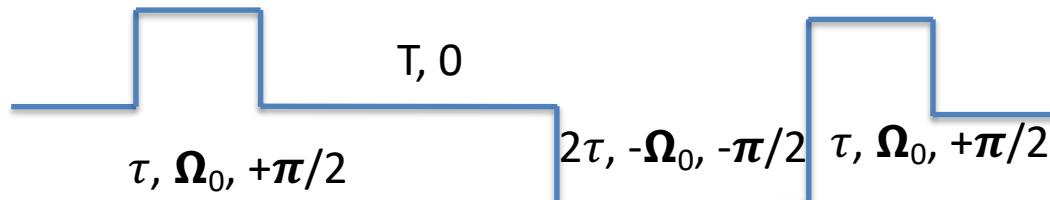
- elimination of the interrogation light shift

Ramsey (phase modulation):



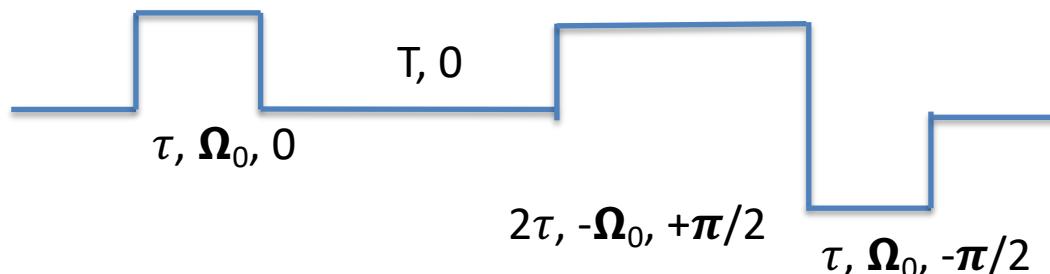
Hyper-Ramsey:

(Yudin, et al., PRA 82, 011804(R) (2010))



Modified Hyper-Ramsey:

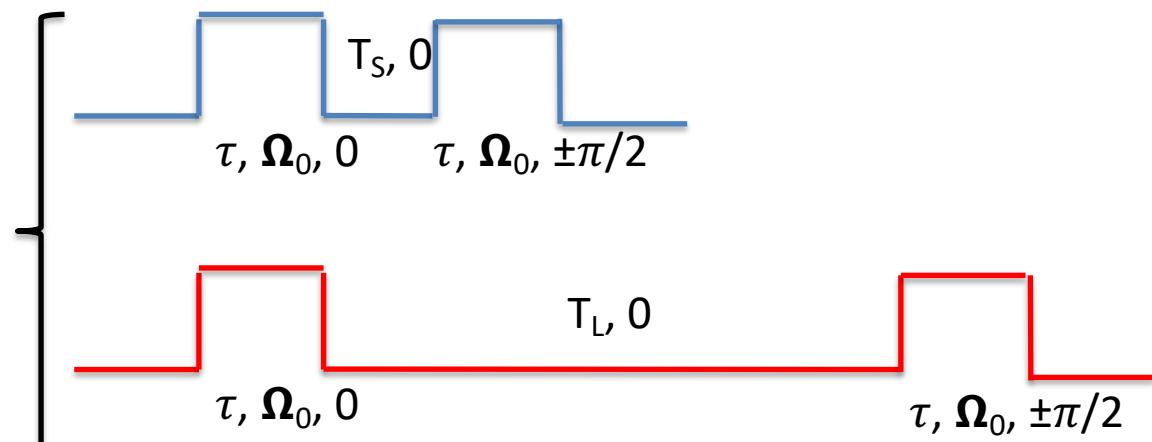
(Hobson, et al., PRA 93, 010501(R) (2016))



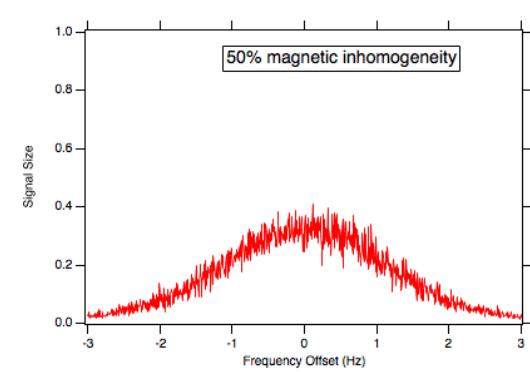
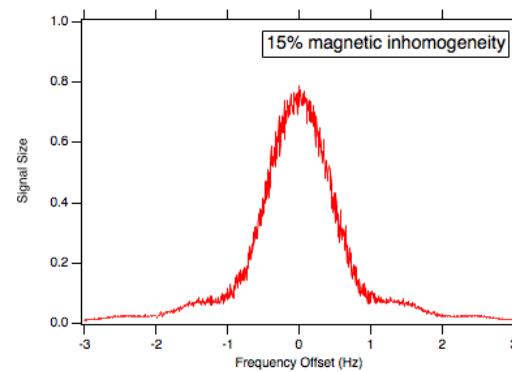
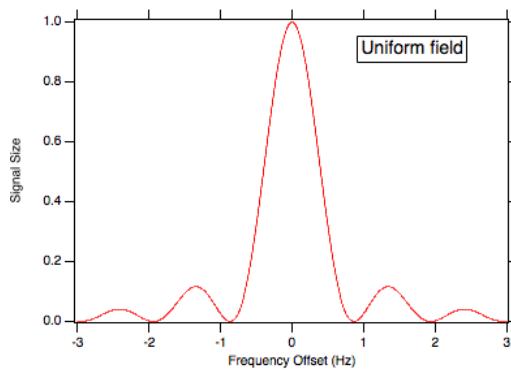
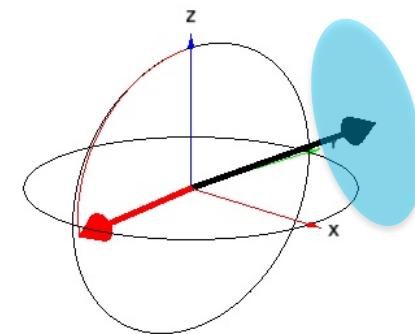
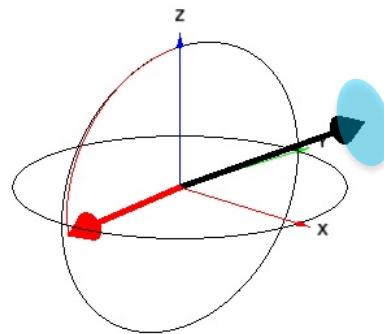
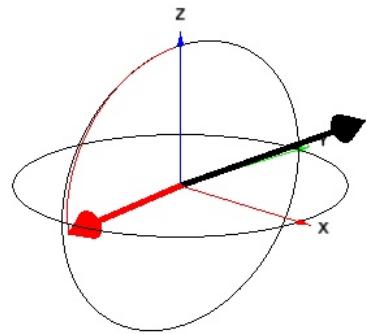
Auto-Balanced Ramsey:

(Samner, et al., ArXiv:1707.02630.v1

[physics.atom-ph] (2017))

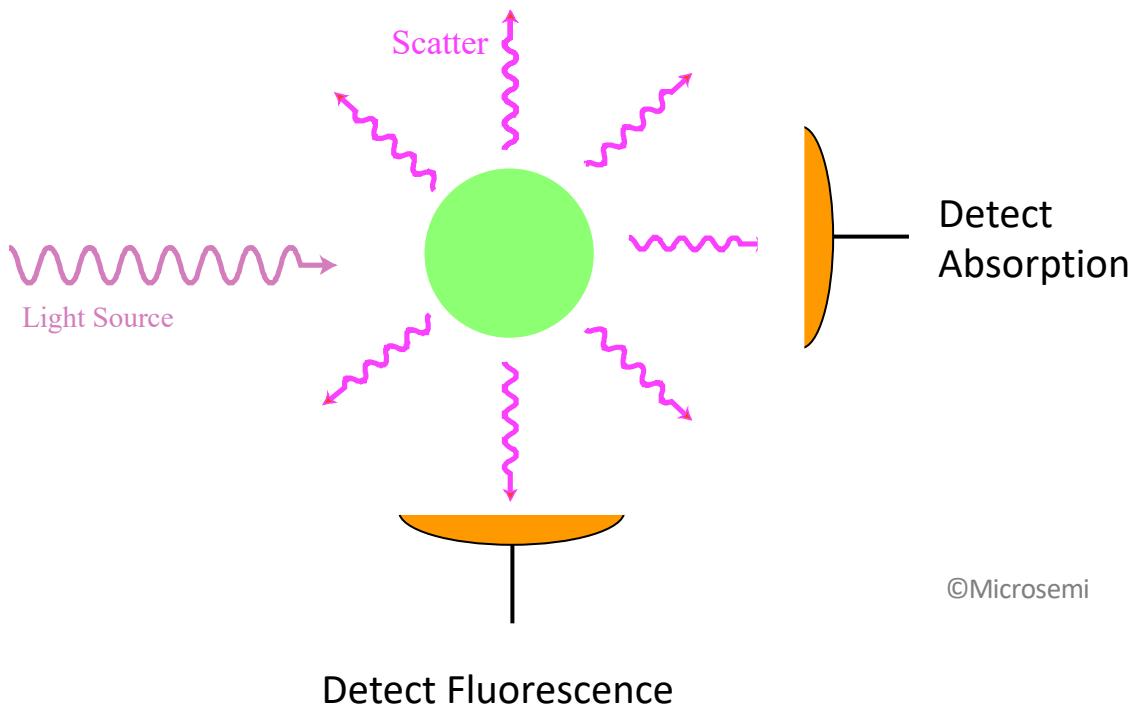


The Bloch Sphere and Decoherence

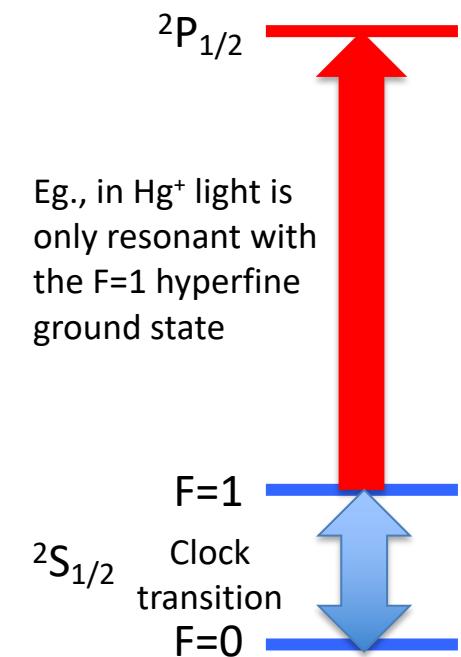


State detection:

Determine what state the atom is in by optical scattering



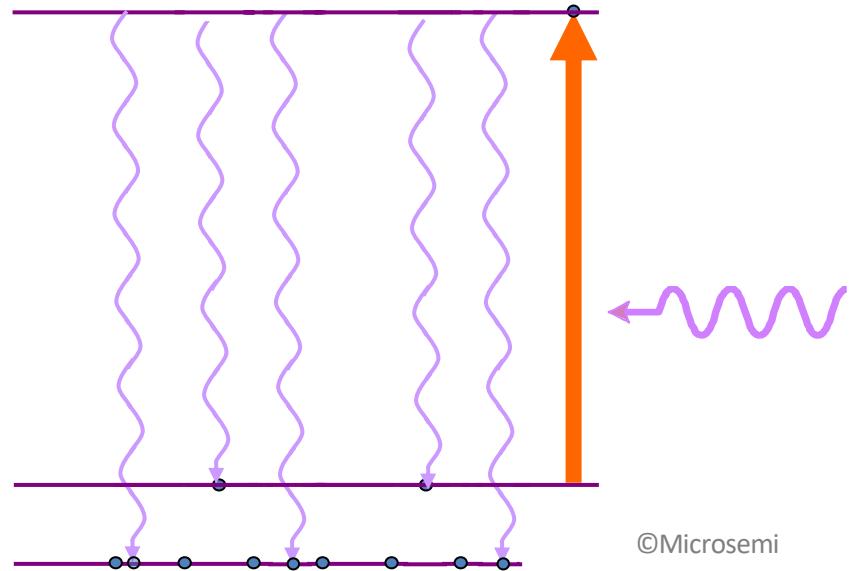
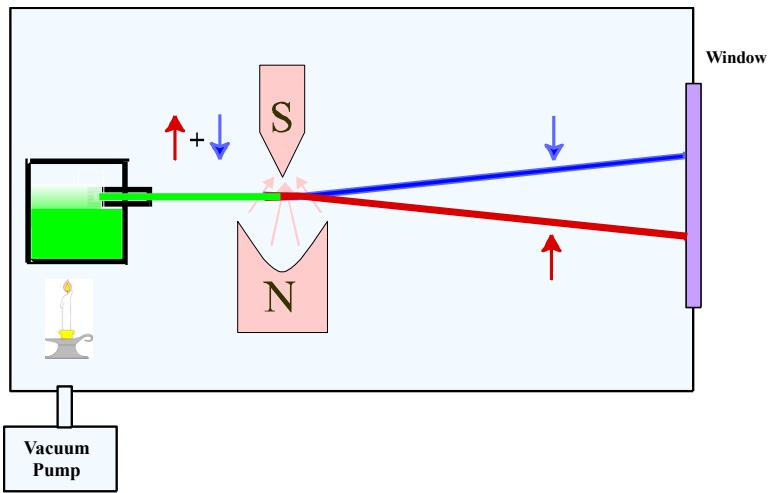
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The physics of key atomic clock techniques

- State selection/detection
- CPT
- Masers
- Laser cooling
- Neutral atom traps – MOT
- Neutral atom traps – Optical lattices
- Ion traps

State selection: magnetic and optical



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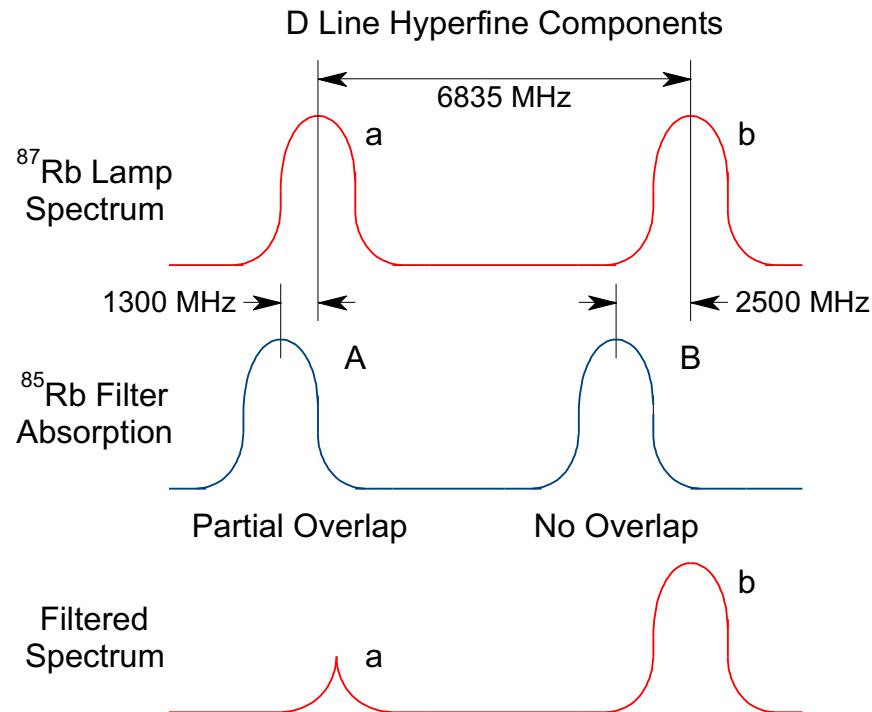
State selection: Hyperfine filtration in rubidium

Problem: a ^{87}Rb lamp will drive **all** transitions

Solution: use a ^{85}Rb filter

“Fortuitous” overlap between the optical absorption lines of the two naturally-occurring isotopes, ^{85}Rb and ^{87}Rb .

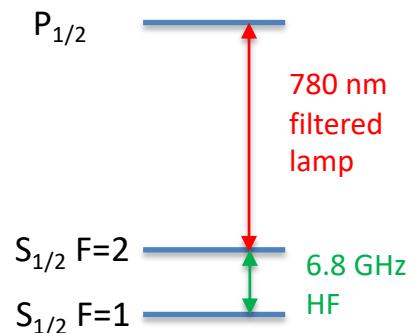
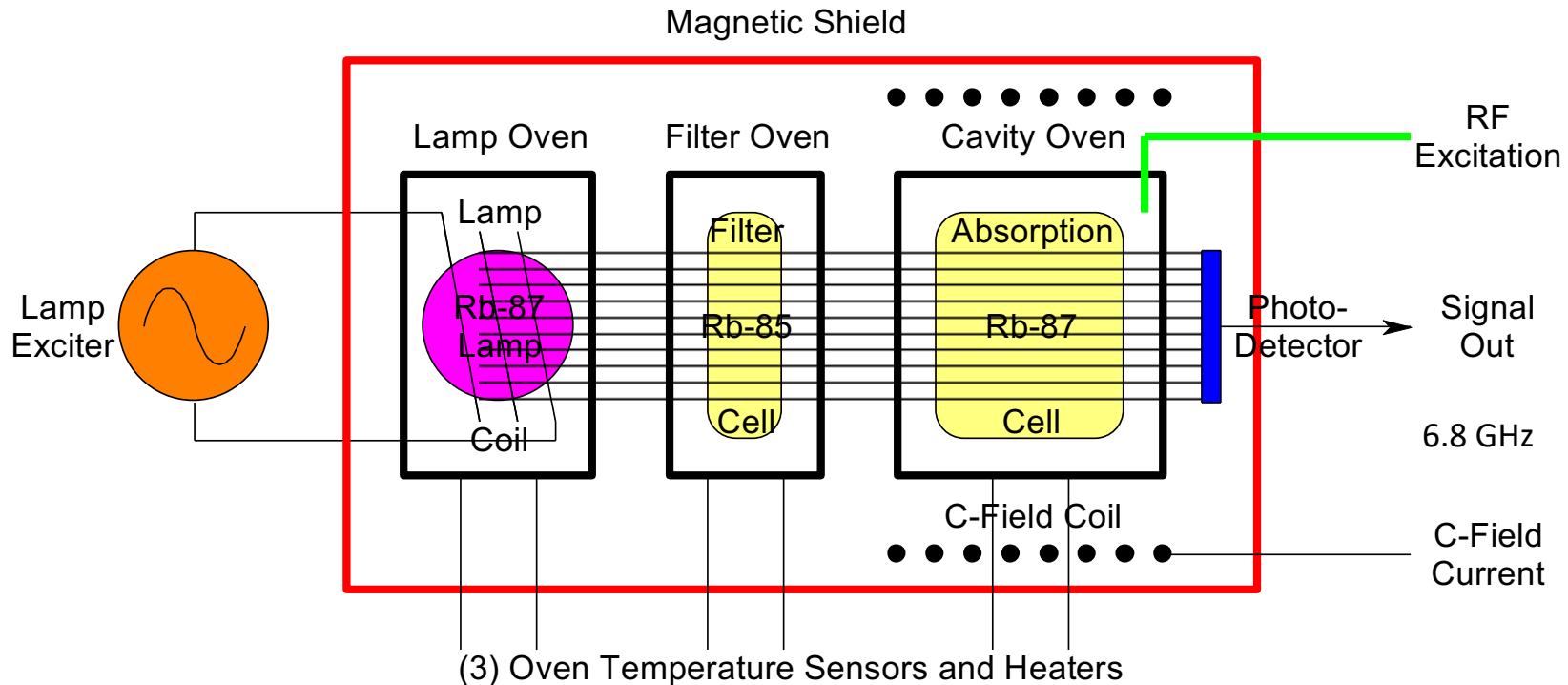
Carver & Alley 1958



Isotopic Filtering of Rubidium 87 D Lines

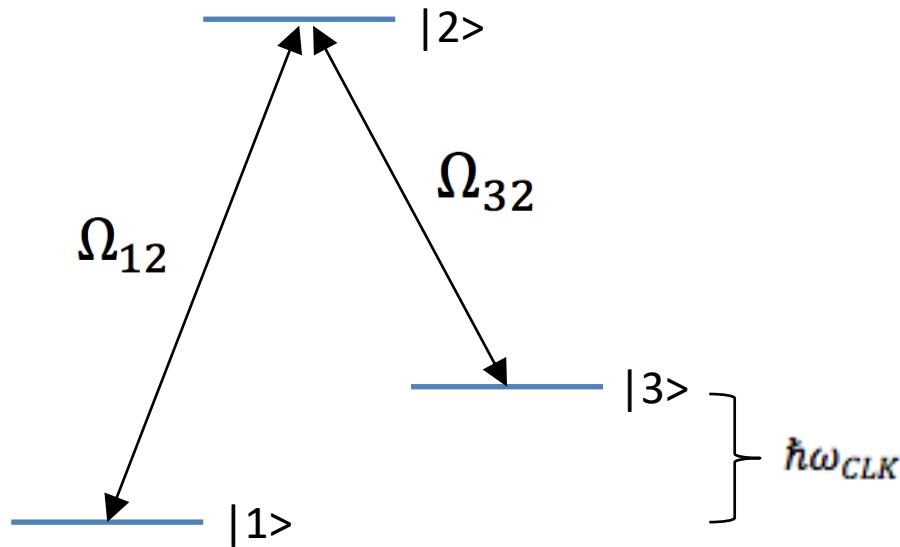
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Rb Gas Cell Physics Package



Coherent Population Trapping

Total Hamiltonian: $H = H_A + H_L + H_I$



New Eigenstates:

$$\psi_1 = \frac{1}{\sqrt{\Omega_{12}^2 + \Omega_{32}^2}} (\Omega_{12}|1\rangle + \Omega_{32}|3\rangle)$$

$$\psi_2 = \frac{1}{\sqrt{\Omega_{12}^2 + \Omega_{32}^2}} (\Omega_{12}|1\rangle - \Omega_{32}|3\rangle)$$

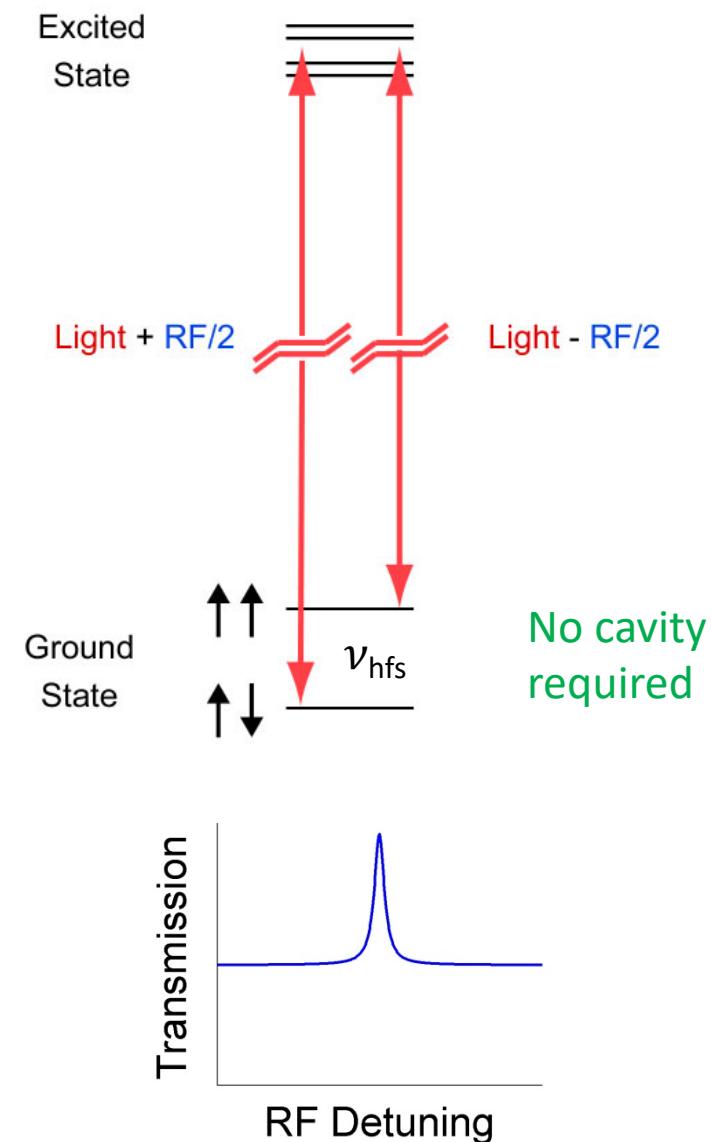
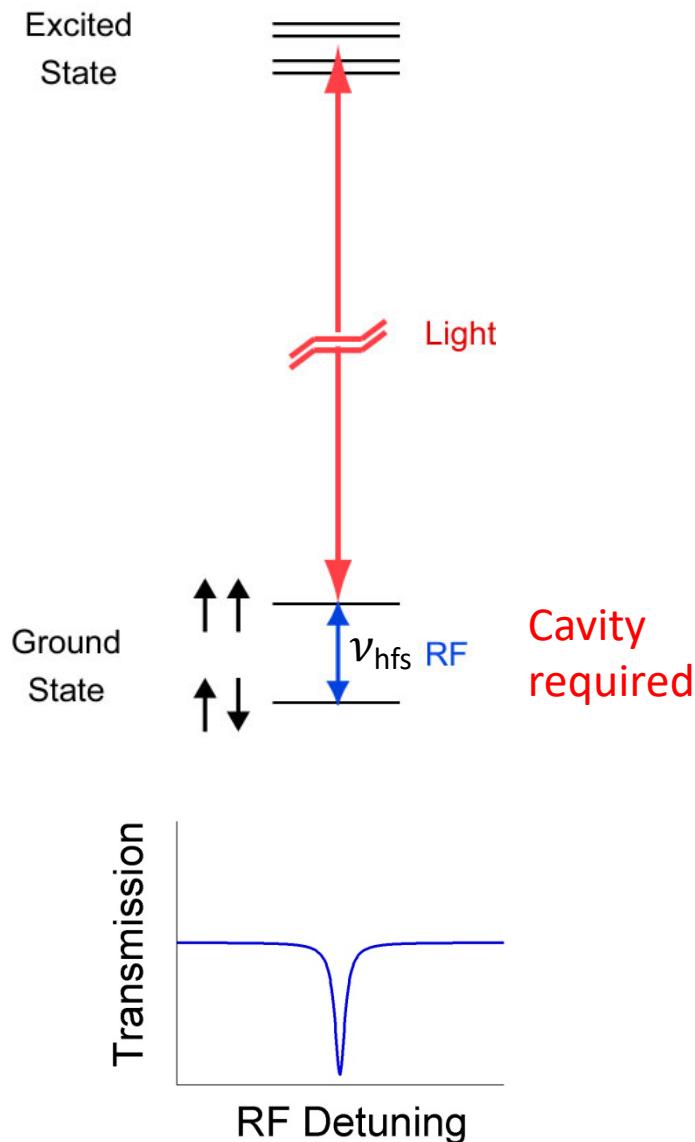
$$\langle 2|H|\psi_1\rangle = d_{12}\Omega_{12} + d_{32}\Omega_{32}$$

$$\langle 2|H|\psi_2\rangle = d_{12}\Omega_{12} - d_{32}\Omega_{32} \rightarrow 0 !$$

With proper intensities ψ_2 is dark (not coupled to $|2\rangle$) and $\omega_{12}-\omega_{32} = \omega_{CLK}$

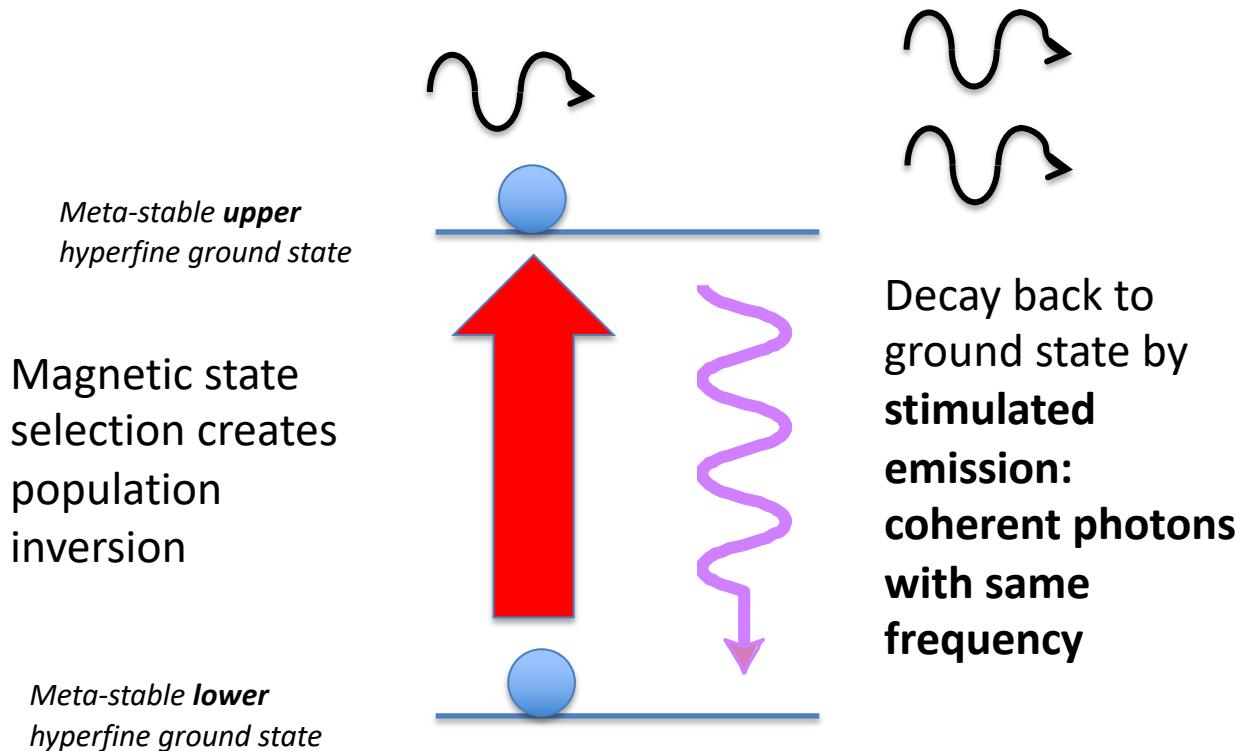
RF vs. CPT Interrogation

“Coherent Population Trapping”:
 $\Delta f = \nu_{\text{hfs}}$ \rightarrow “dark state” \rightarrow transmission



Microwave Amplification of Stimulated Emission Radiation: The Maser

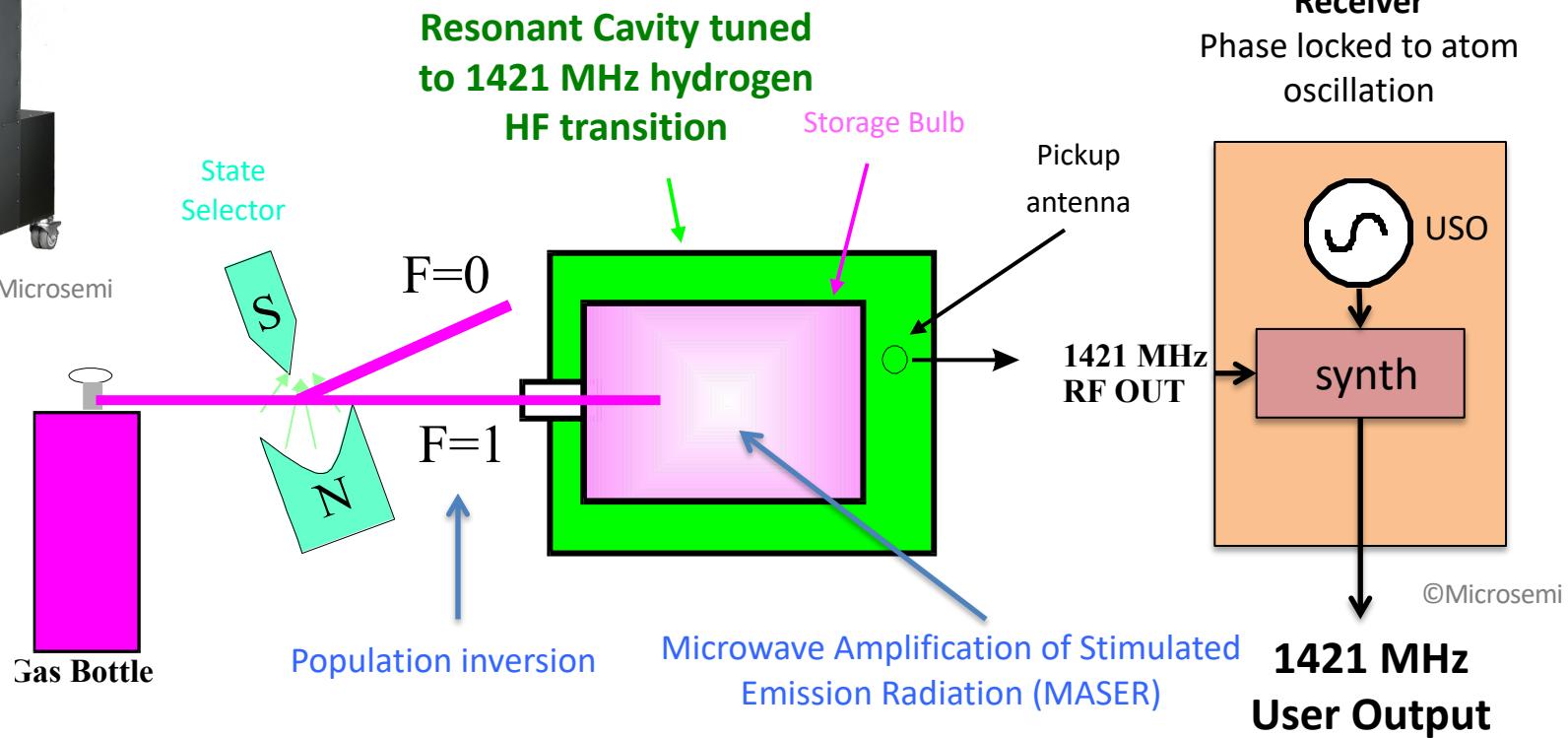
(Microwave counterpart to laser)



Active Hydrogen Maser

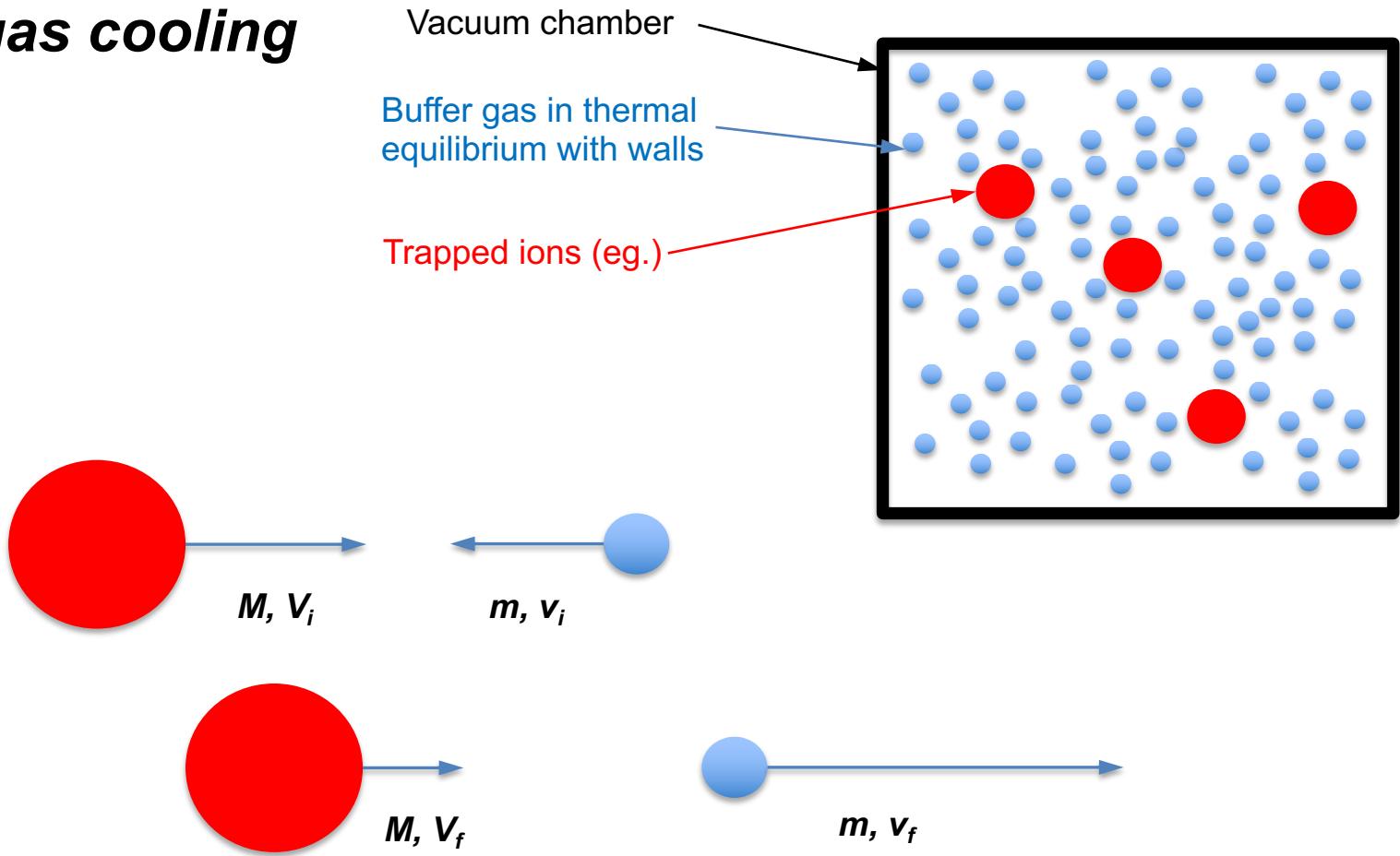


Microsemi
Model
MHM2010



- Active Device analogous to laser
- Excellent short term stability (10^{-13} at 1 s, 10^{-15} at 1000 s)
- Drifts with cavity/wall properties: 10^{-15} /day typical
- Lamb-Dicke Confinement: no 1st order Doppler

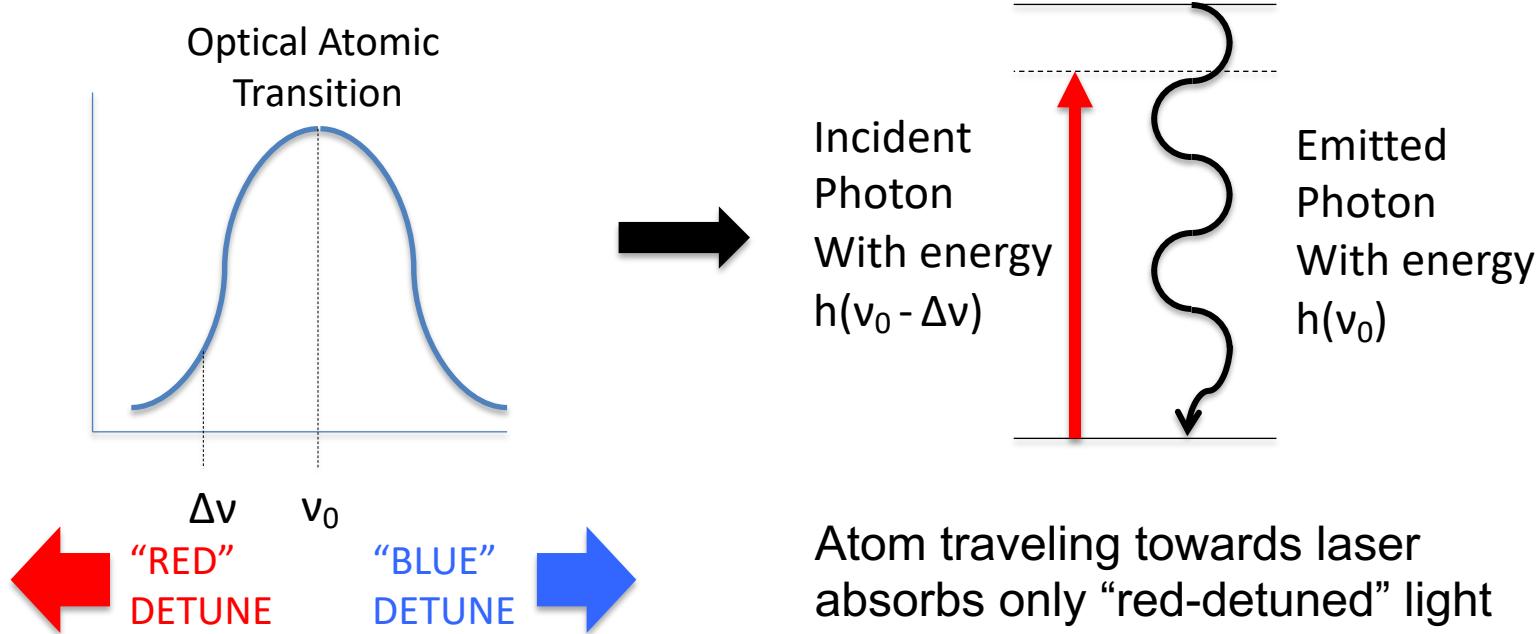
Buffer gas cooling



Conservation of momentum and energy => (to be completed)

Laser Cooling

(S. Chu, C. Cohen-Tannoudji, and W.D. Phillips Nobel prize 1997)

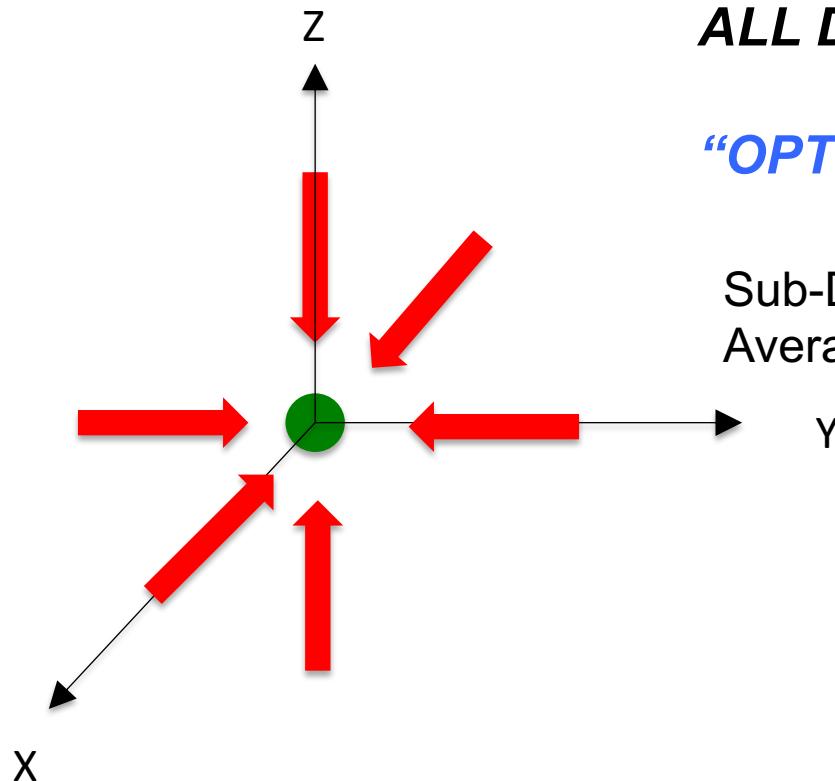


Net effect:

- Atom emits more energy than it absorbs
- atom slows down in 1D

Laser Cooling

Now add “red-detuned” lasers in all three directions



ALL DIRECTIONS SLOWED BY LIGHT:
“OPTICAL MOLASSES”

Sub-Doppler cooling $\rightarrow \sim 1 \text{ } \mu\text{K}$
Average velocity $\sim 1 \text{ cm/s}$



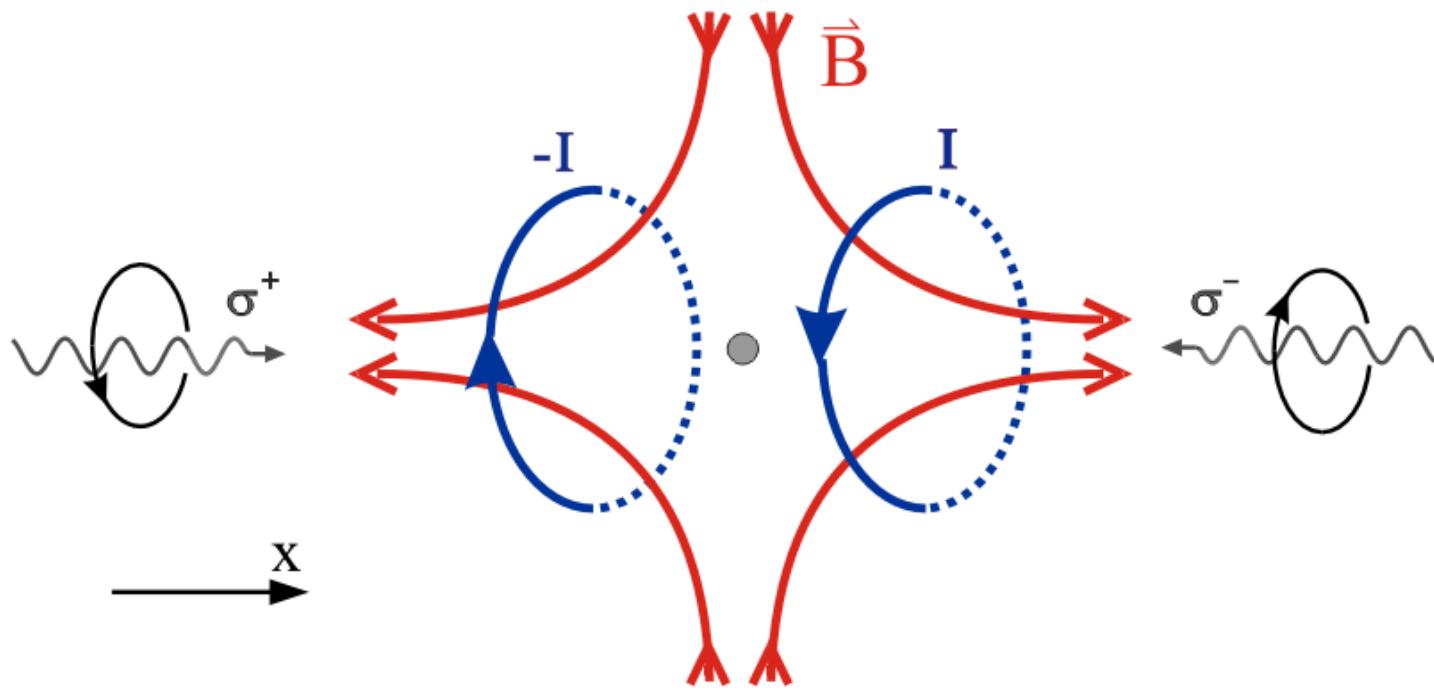
USNO

Note: by unbalancing opposing beams, can move atoms

Laser Cooling: Motivation

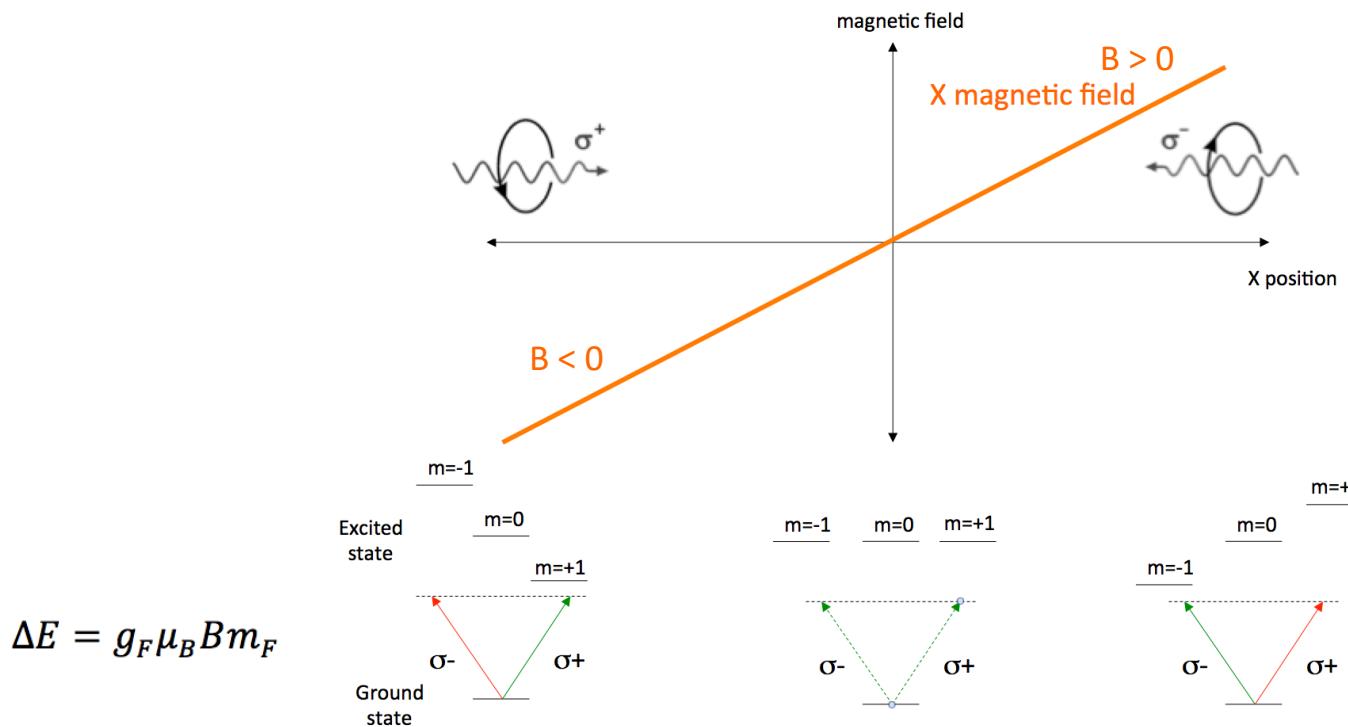
- Atom confinement for $O(s)$ instead of $O(ms)$
- Reduce Doppler effect
- Enables trapping and in-situ interrogation
- Enables moving atom ensembles over macroscopic distances
 - Atomic fountains

Neutral Atom Trapping – The MOT



Magnetic Field + Circularly Polarized Light
= Position-dependent restoring force
= Magneto-Optic Trap (MOT) !

Neutral Atom Trapping – The MOT



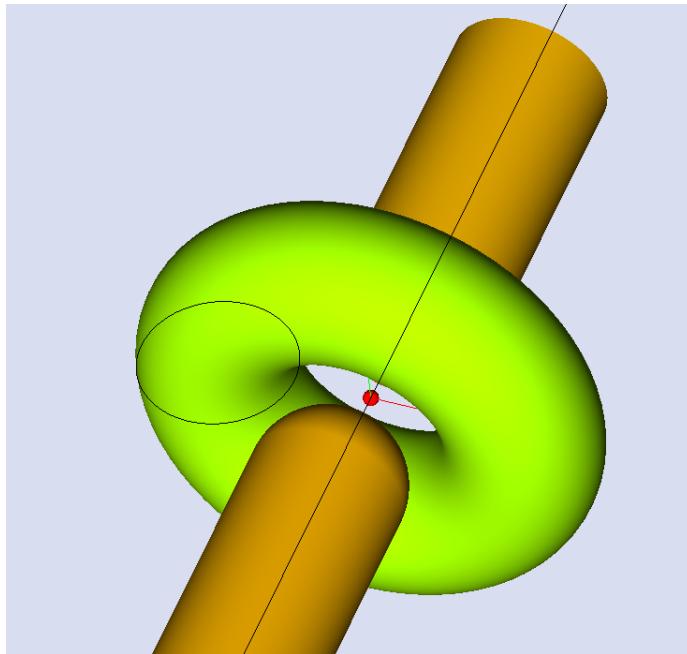
- Combine detuning, polarization, Zeeman shift: **create a position-dependent restoring force**
- Circular polarization drives only $\Delta m = \pm 1$ Zeeman transitions
- Strong field away from geometric center shifts transition into resonance with laser
 - => **light restoring pressure proportional to distance from center**
- Weak field in center: laser off resonance – little or no interaction with atoms
- Atoms localized and laser-cooled in 1D
- Extend to 3D with additional beams: Magneto Optical Trap (MOT)

Neutral atom trapping: optical lattices

(to be completed)

Ion Trapping: *The quadrupole Paul Trap*

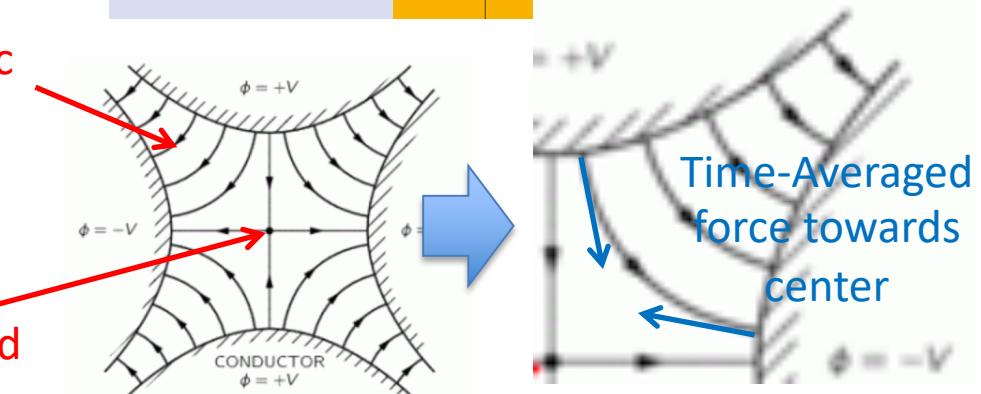
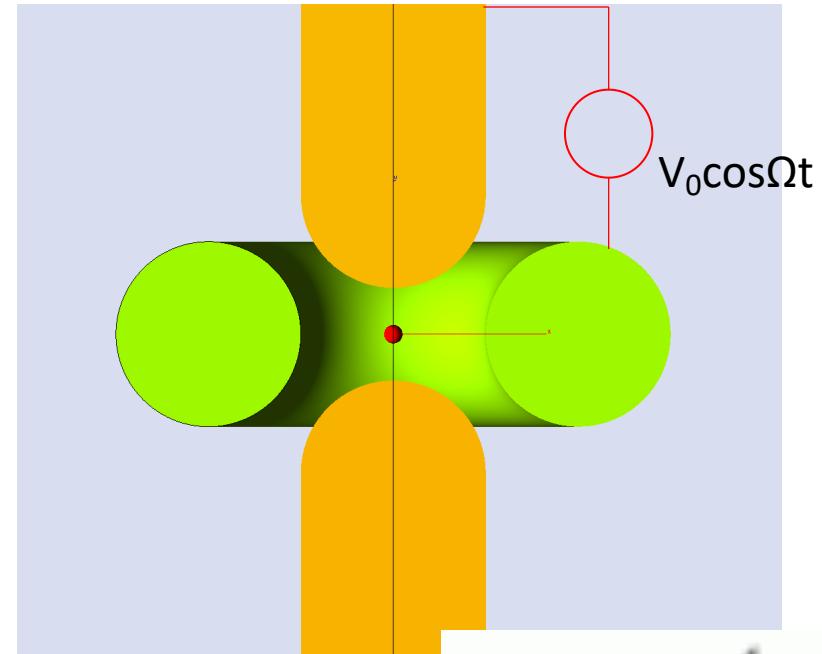
(Hans Dehmelt and Wolfgang Paul, Nobel Prize, 1989*)



Typical well depth: several eV
(room temperature is 1/40 eV)

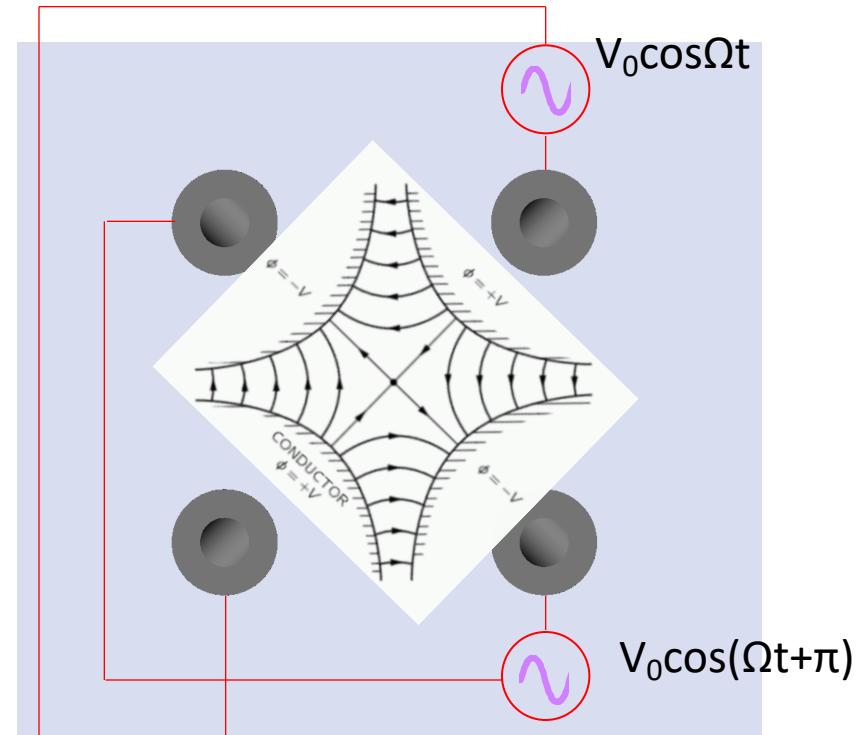
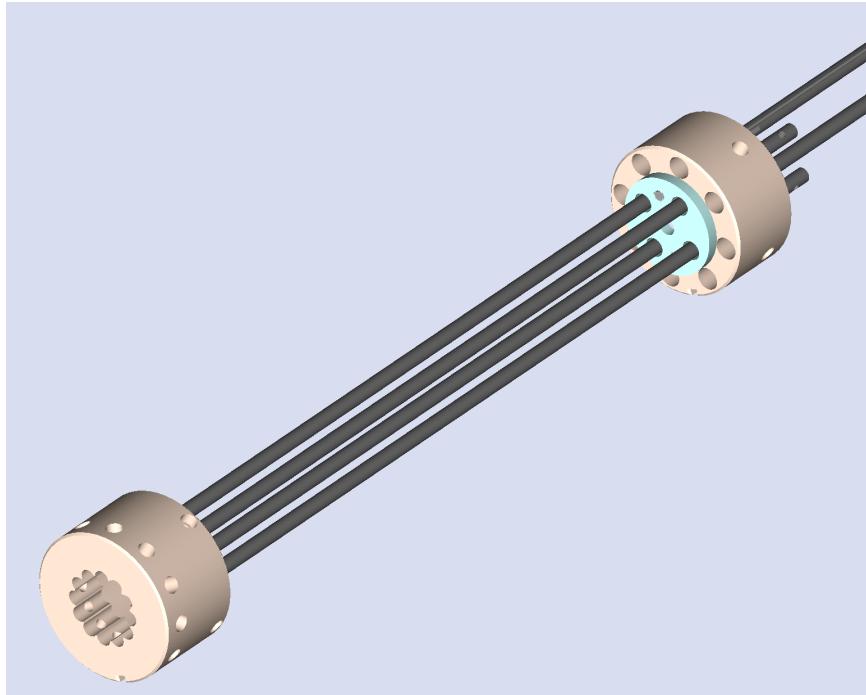
RF electric field

Ions seek
region of
weakest field



*Normal Ramsey shared this Nobel prize for his invention of the method of separated oscillatory fields

*Ion Trapping: The quadrupole linear ion trap**



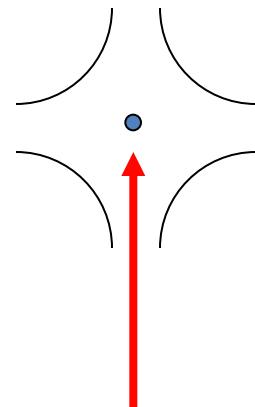
Number of ions scales up linearly

$$N_{lin} = \frac{3}{5} \left(\frac{L}{R_{sph}} \right) N_{sph}$$

Ion Trapping: The Multi-pole linear ion trap

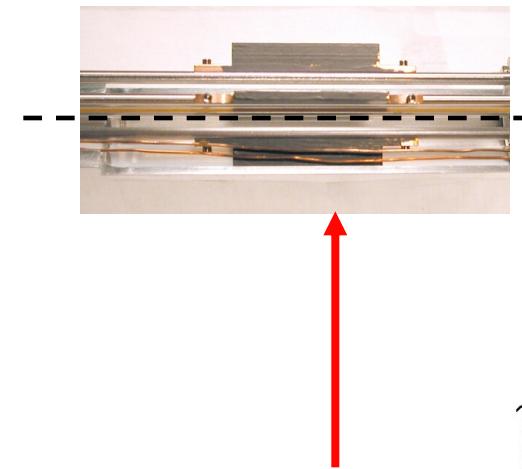
Multi-pole (12)
RF Trap

Spherical
Quadrupole
RF Trap



Field-free region at one
point in center of trap

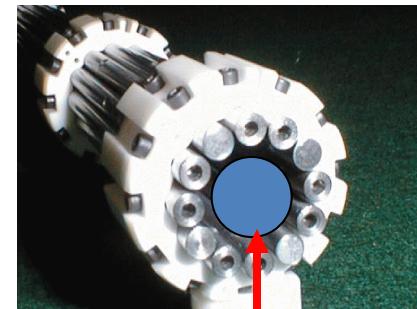
Linear
Quadrupole
RF Trap



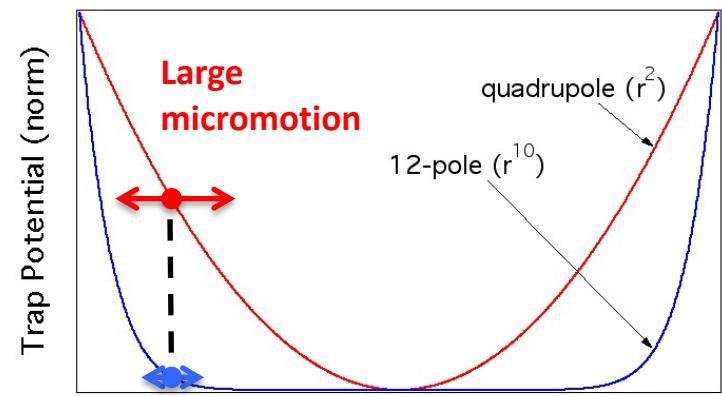
Field-free
region
on a line

H.G. Dehmelt, Bull. Am.
Phys. Soc. 18, 1521 (1973)

J.D. Prestage, G.J. Dick,
and L. Maleki, J. Appl.
Phys. 66, 1013 (1989)



Field- "free"
Region in a
volume



Small
micromotion
radial position (norm)
J.D. Prestage, R.L. Tjoelker,
and L. Maleki, Proceedings
of the 1999 Joint EFTF-FCS

Ion trapping: simulations

Single ion equations of motion: The Mathieu equations

$$\ddot{x} + \frac{e}{mr_0^2} (U - V \cos \Omega t) = 0, \quad \ddot{y} - \frac{e}{mr_0^2} (U - V \cos \Omega t)$$

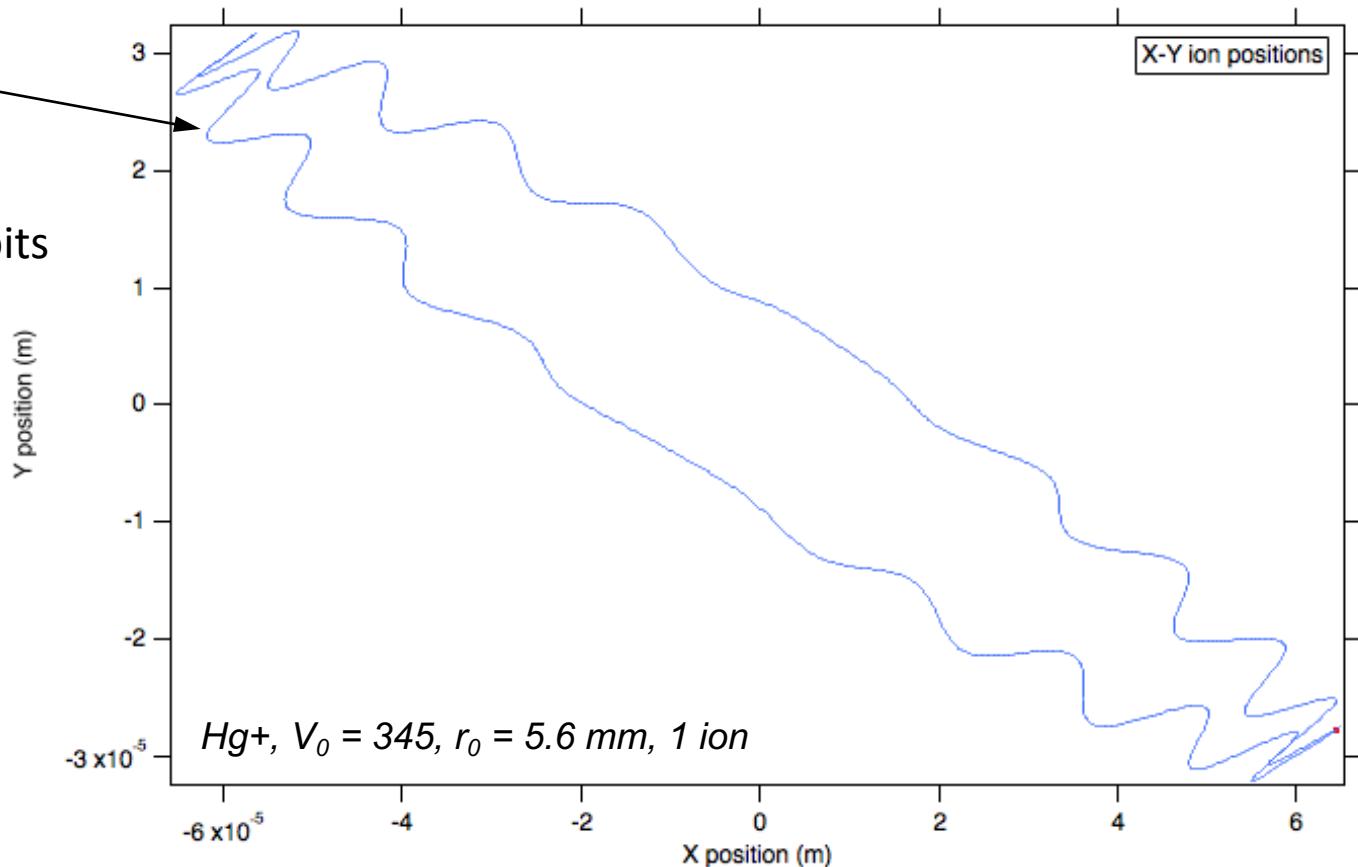
Micromotion

- $f_m = f_{\text{trap-drive}}$

Secular motion = orbits

- $f_s \sim 0.1 - 1 \text{ MHz}$

Movie!

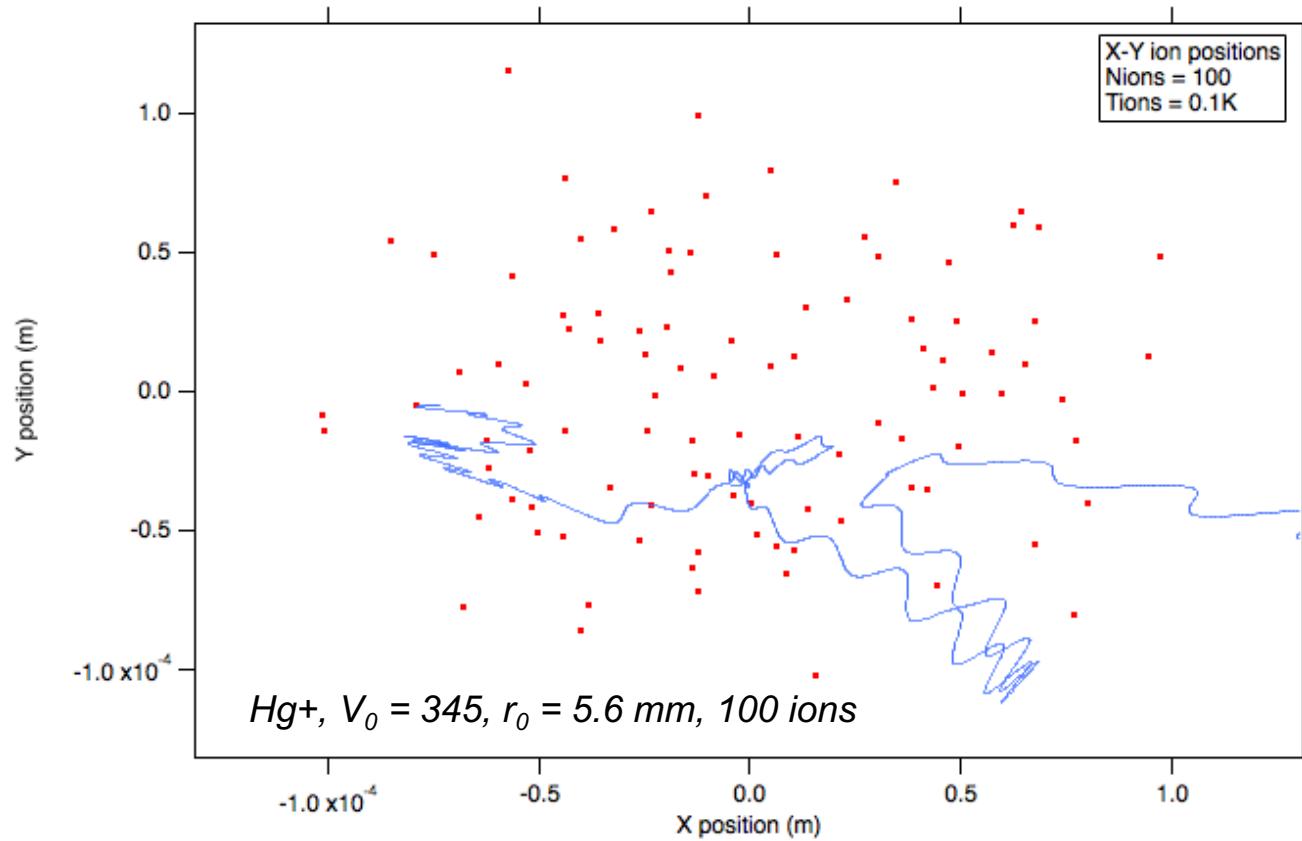


Ion trapping: simulations

Many ions - add Coulomb interactions

$$\ddot{x}_i + \frac{e}{mr_0^2}(U - V\cos\Omega t) - \sum_{j \neq i} \frac{1}{4\pi\epsilon_0 m} \frac{e^2}{x_{ij}^2} = 0$$

Movie!

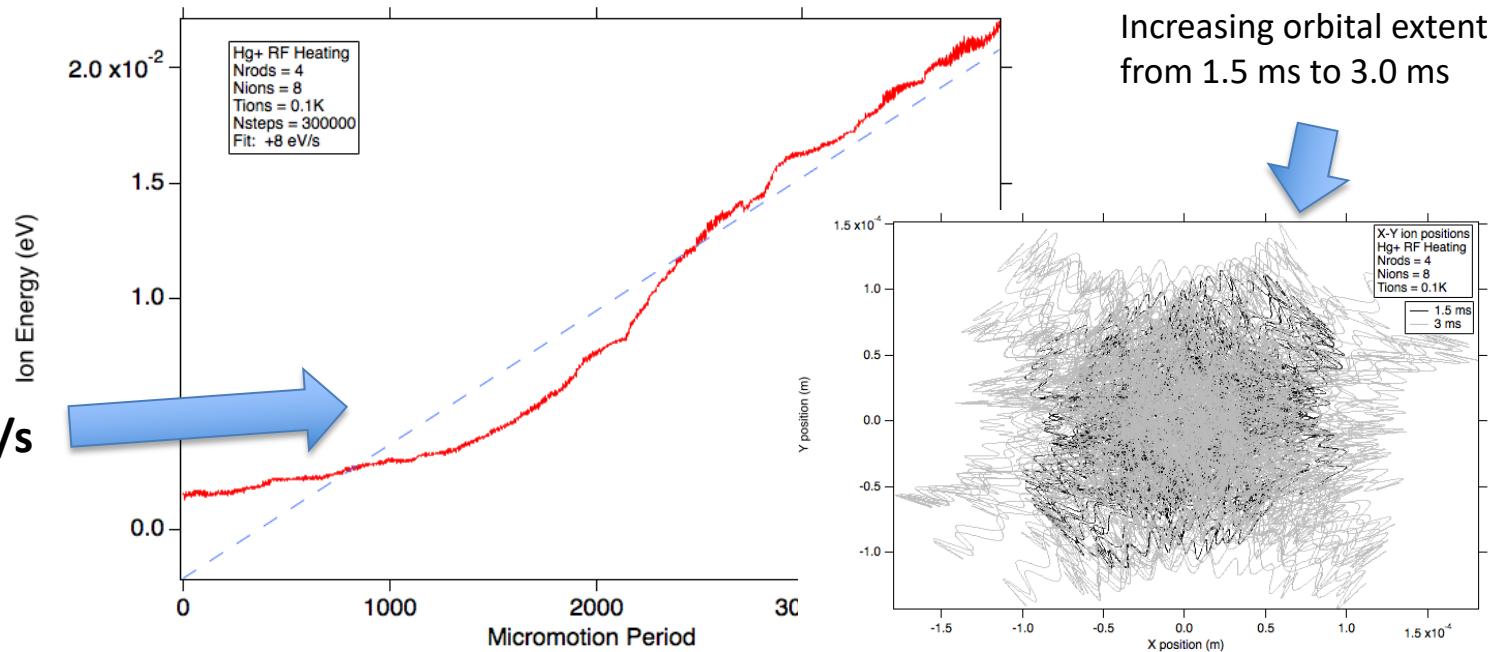


Ion trapping: simulations

Many ions – RF heating

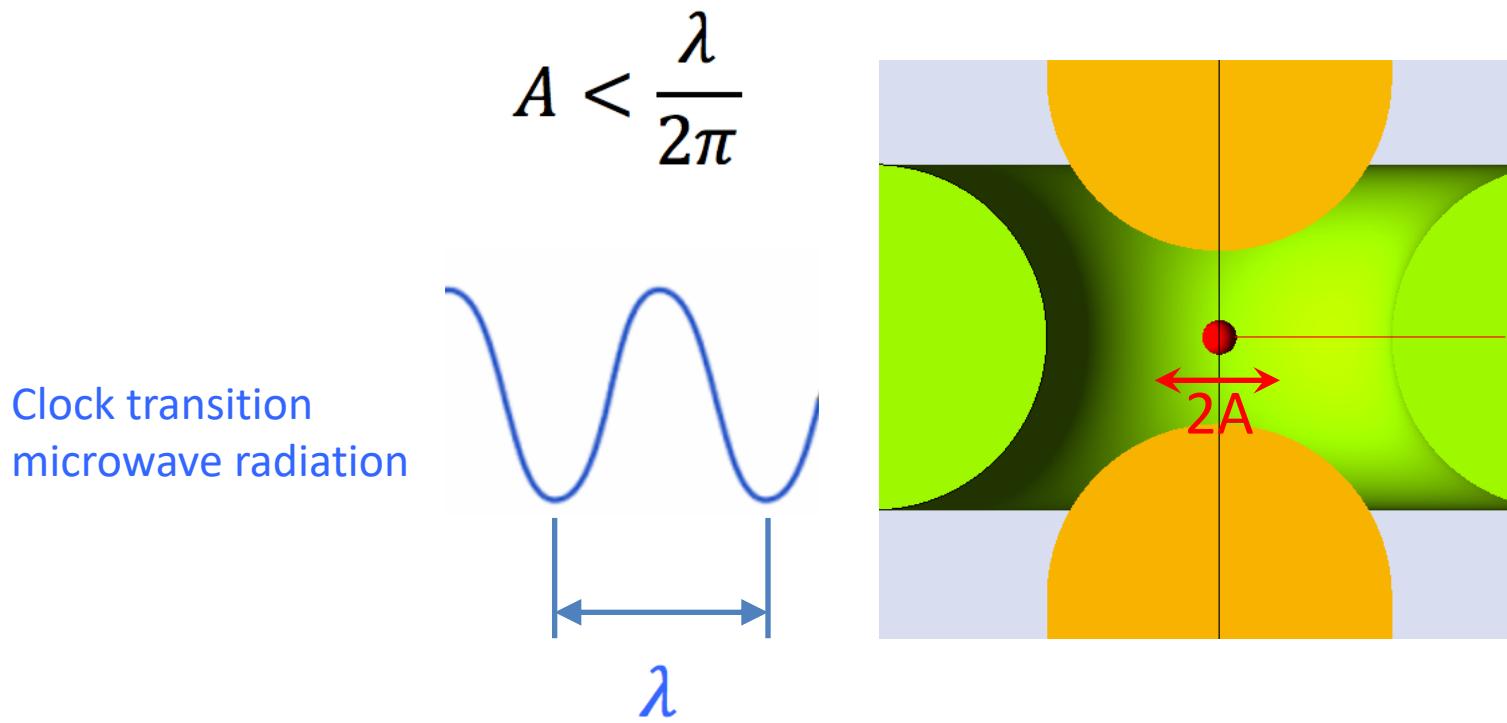
How does RF heating work?

- Only present when there are more than one ion experiencing collisions
- Consider collision where $v_{ion} = 0$ (*Cetina, et al., PRL 109, 253201 (2012)*)
- $V_{ion} = v_{sec} + v_{micro} \Rightarrow v_{sec} = -v_{micro}$
- Conservative potential: $v_{ion} = v_{sec} = 0$
- RF trap field has done work on the ion during the collision



Ion trapping: motivation

- Eliminate “end-to-end” effects
- Can extend confinement almost indefinitely
- very long interrogation times possible
- No wall effect !
- Eliminate the first order Doppler effect: **Lamb-Dicke Confinement**



Key frequency shift effects in atomic clocks

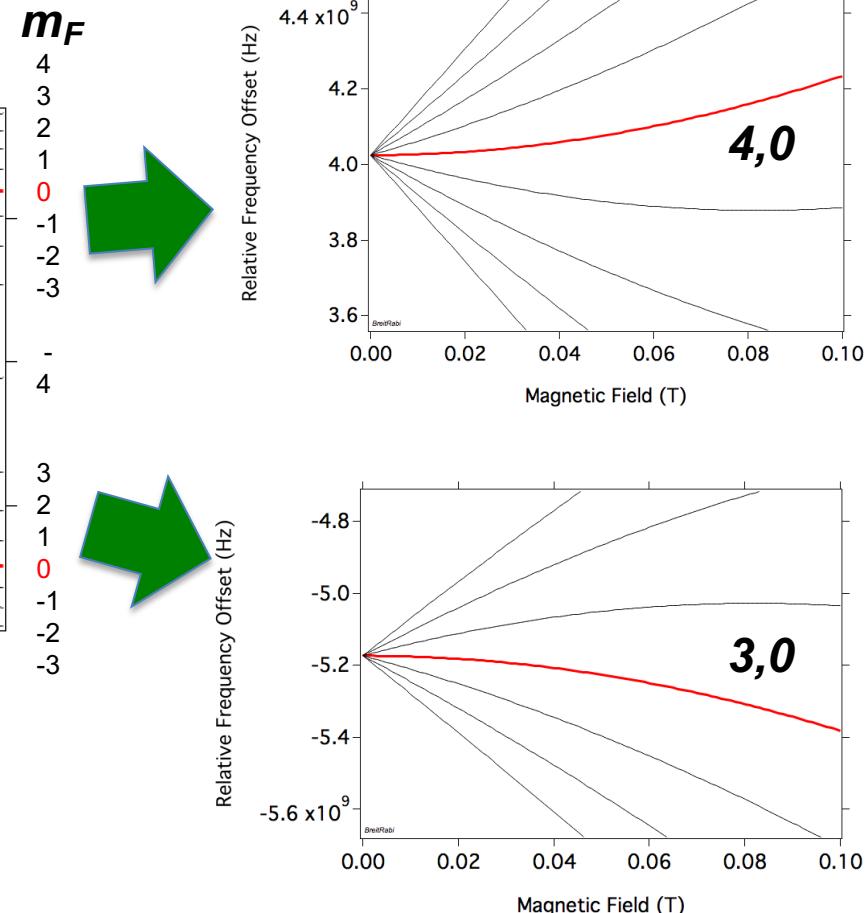
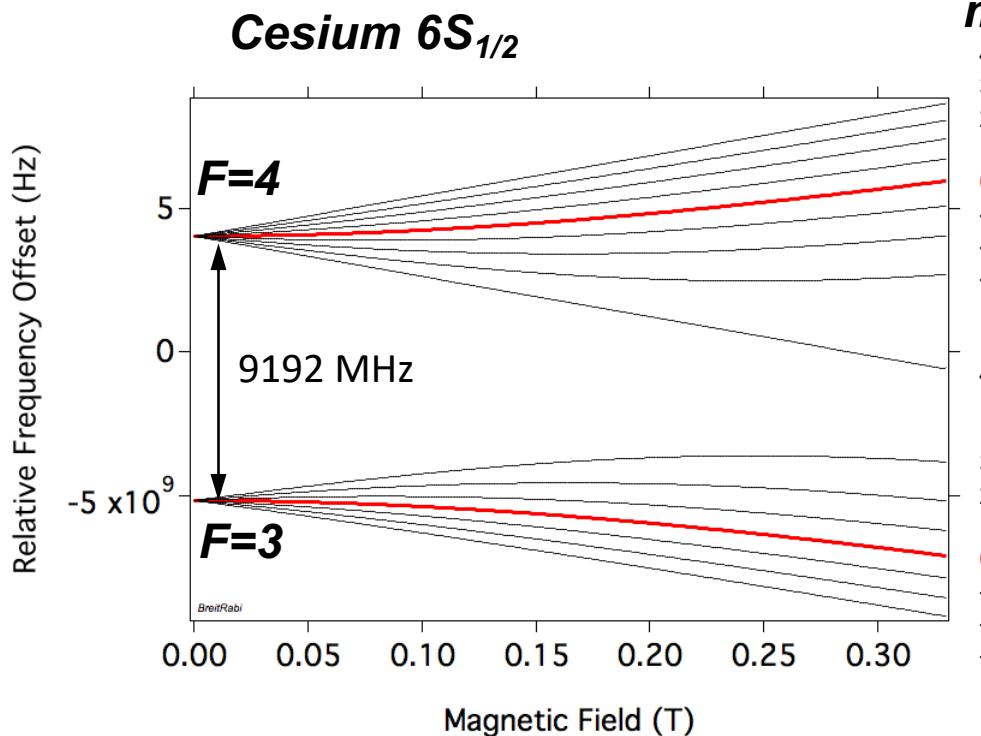
- Magnetic
- Electric
- Collisional
- Doppler
- Gravitational

Magnetic effects: Zeeman shifts (1st and 2nd order)*

The Breit-Rabi formula:

$$\Delta E(F, M) = -\frac{h\Delta\nu}{2(2I + 1)} - g'_I \mu_B B M \pm \frac{1}{2} h\Delta\nu \times \left\{ 1 + \frac{4M}{2I + 1} x + x^2 \right\}^{1/2}$$

$$x = \frac{(g_J + g'_I)\mu_B B}{h\Delta\nu}$$



*See "Elementary Atomic Structure", 2nd Edition, by G.K. Woodgate, Oxford University Press, eq. 9.80, p. 193

Electric effects: AC Stark (light) shift to first order

$$\Delta\nu^{E1} = (\Delta\kappa^s + \Delta\kappa^v m_F \xi \vec{e}_k \cdot \vec{e}_B + \Delta\kappa^t \beta) U_0$$

Field magnitude:

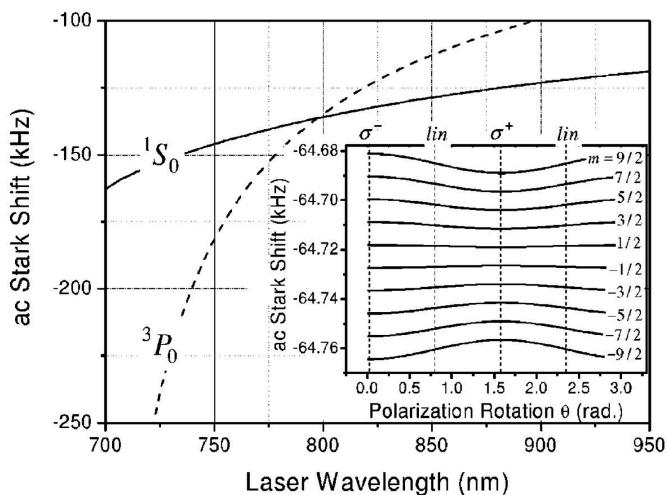
- Scalar light shift
- Magic wavelength¹

Field alignment:

- Vector light shift²
- Alignment of k rel. to B

Field orientation:

- tensor light shift²
- Orientation of E rel. to B
- Magic angle



2-photon:

$$\Delta\nu^{hyper} = \Delta\kappa^h (U_0/E_r)^2$$

Higher pole:

$$\Delta\nu^{M1/E2} = \Delta\kappa^{M1/E2} \sqrt{U_0}$$

1. H. Katori, et al., PRL **91**, 173005 (2003)
2. P.G. Westergaard, , Phys. Rev. Lett. **106**, 210801 (2011)

Electric effects: AC Stark (light) shift to higher order

Hyperpolarizability

- 2 photon effect
- Can't zero it out
- Operate with low trap field

$$\Delta\nu^{hyper} = \Delta\kappa^h (U_0/E_r)^2$$

Higher trap field poles

- Magnetic dipole or electric quadrupole
- Also can't zero out
- Operate with low trap field

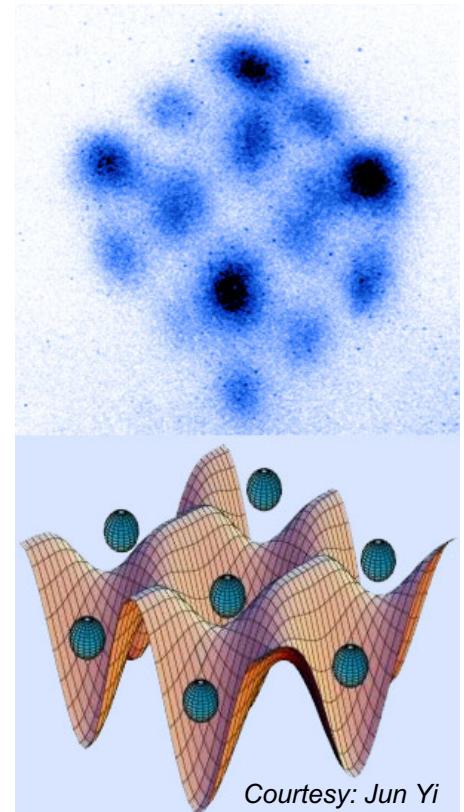
$$\Delta\nu^{M1/E2} = \Delta\kappa^{M1/E2} \sqrt{U_0}$$

*P.G. Westergaard, , Phys. Rev. Lett. **106**, 210801 (2011)*

Collision shifts – ultra-cold

- S-wave collisions vs. room temperature
- Glancing collisions!
- Bosonic vs. Fermionic species

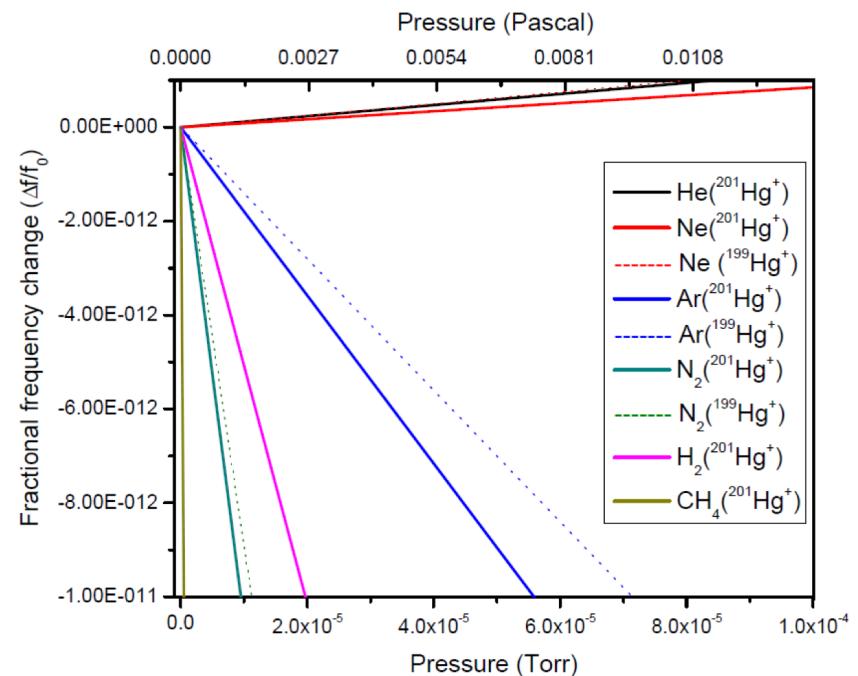
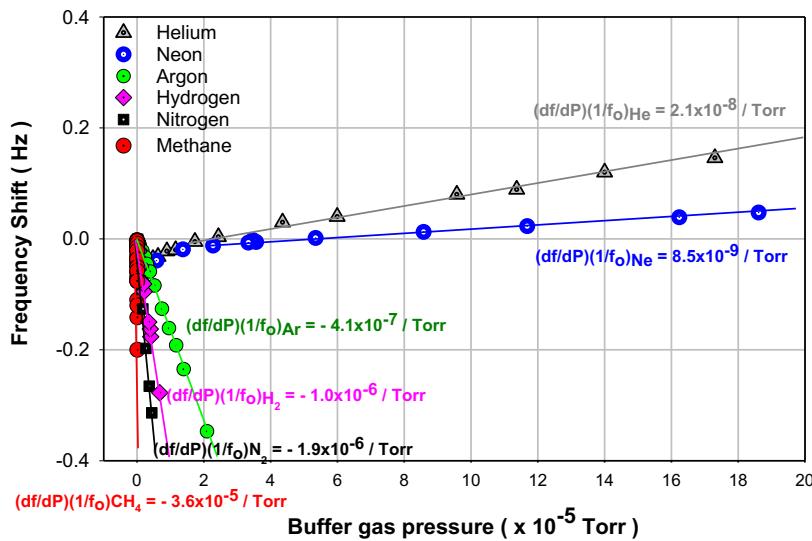
(to be completed)



Collision shifts – room temperature

$$\frac{\Delta f}{f} = K_i P_i$$

eg., in Hg+ (hyperfine), $K_{H_2} = 3e-6/\text{torr}$, $K_{CH_4} = 1e-5/\text{torr}$, $K_{He} = 1e-8/\text{torr}$
 $\Rightarrow \text{If } \Delta P_{CH_4} = 1e-10 \text{ torr}, \Delta f/f = 1e-15 !!$



S.K. Chung, et al., Proc. IEEE IFCS (2004)

L. Yi, et al., Proc. IEEE IFCS (2012)

Doppler shifts – 1st and 2nd order

- First order:
 - Lamb-Dicke confinement
- Second order:
 - Laser cool to zero point
 - Multi-pole traps
 - Compensation schemes

$$\frac{\Delta f}{f} = \frac{v}{c}$$

$$\frac{\Delta f}{f} = -\frac{3k_B T}{2mc^2} \left(1 + \frac{2}{3} \frac{1}{k-1} \right)$$

(to be completed)

Gravitational shifts

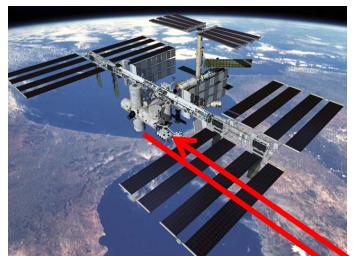
- Gravitational red shift

(to be completed)

Overview of fundamental physics tests with atomic clocks

- Fundamental constants
- Compare different atomic species
- Compare mechanical and atomic standards
- Isotopic effects
- Gravitational waves
- VLBI
- Dark energy and precision ranging
- Occultation studies

Gravitational red shift: Atomic Clock Ensemble in Space Mission (ACES)- JPL reference



ISS clocks

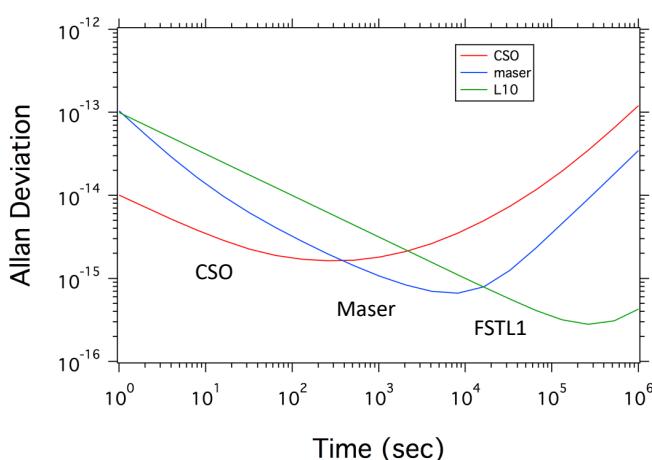
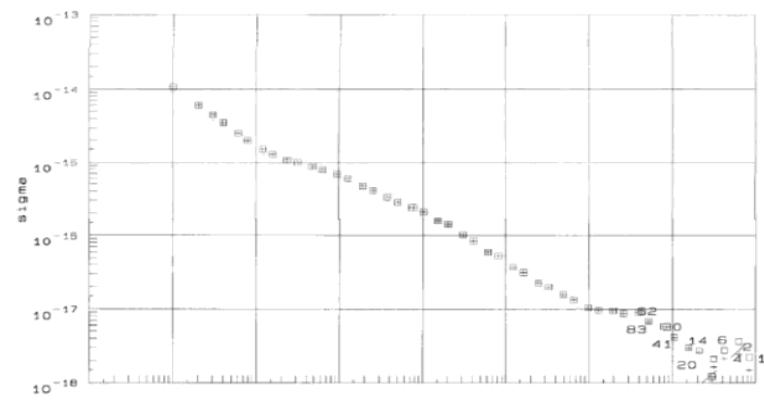


ACES
ground terminal



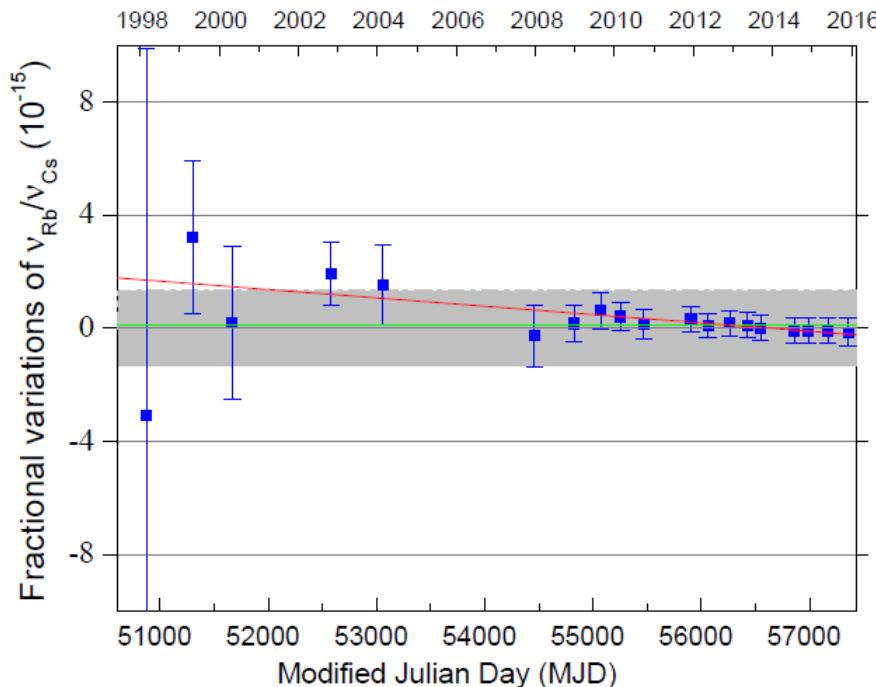
Power & Telemetry
100 MHz, 1pps
Loopback monitor
~100 m

Span: 140701.070817 to 150722.145018, 33378121 s
Here: 140701.070817 to 150722.145018, 33378121 s
0 33378121
Est. drift: -5.401E-20/d Sigma: 2.072E-20 Gross □ Net □



H-maser LO – Hg+ reference

Rb/Cs: search for time variation in fundamental constants



Phys. Rev. Lett. 109, 080801 (2012)

Weighted least-squares fit to a line

$$\frac{d}{dt} \ln\left(\frac{\nu_{Rb}}{\nu_{Cs}}\right) = (-10.7 \pm 4.9) \times 10^{-17} \text{ yr}^{-1}$$

⇒ limit on a potential variation of fundamental constants :

With QED calculations: *J. Prestage, et al., PRL (1995), V. Dzuba, et al., PRA (1999)*

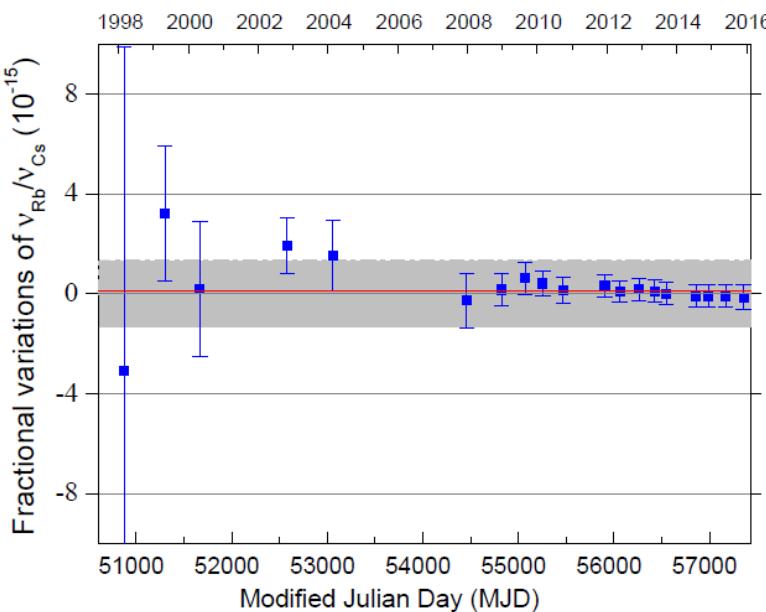
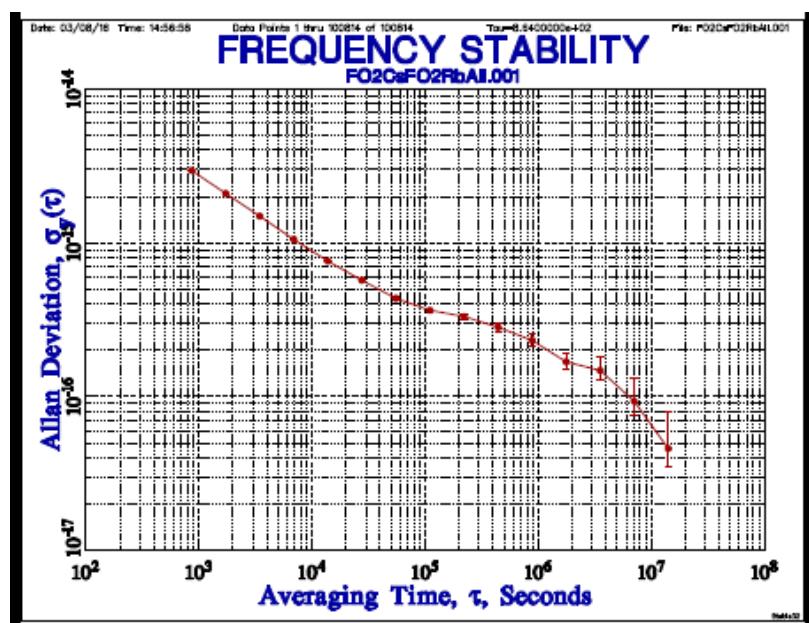
$$\frac{d}{dt} \ln\left(\frac{g_{Rb}}{g_{Cs}} \alpha^{-0.49}\right) = (-10.7 \pm 4.9) \times 10^{-17} \text{ yr}^{-1}$$

With QCD calculations: *T.H. Dinh, et al., PRA79 (2009)*

$$\frac{d}{dt} \ln[\alpha^{-0.49} (m_q / \Lambda_{QCD})^{-0.021}] = (-10.7 \pm 4.9) \times 10^{-17} \text{ yr}^{-1}$$

Fundamental Physics with Rb/Cs fountains (SYRTE)

- 16 years of ^{87}Rb ground state hyperfine frequency measurements against Cs : FO2-Rb against FO1 or FOM, and since 2009 against FO2-Cs operated simultaneously.
- Feb. to Aug. 2012 measurement
 $6\ 834\ 682\ 610.904\ 312\ (3)\ \text{Hz} (\pm 4.4 \times 10^{-16})$
 ⇒ recommended value of Rb hf frequency

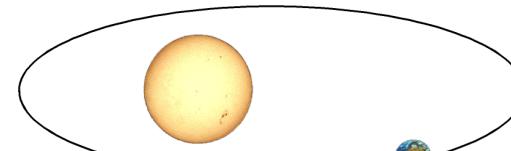
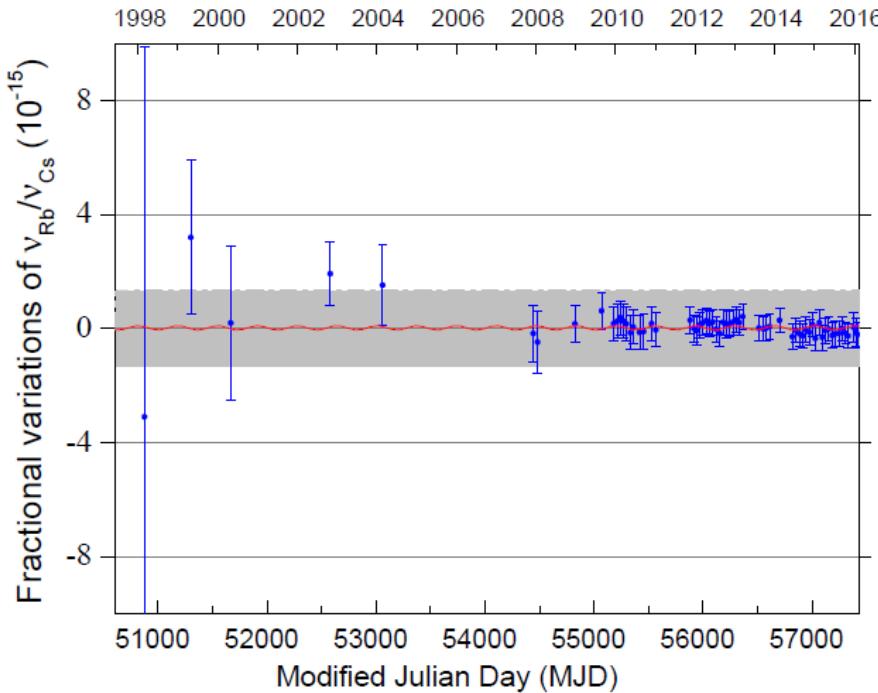


Phys. Rev. Lett. 109, 080801 (2012)

FO2Cs – FO2Rb long term comparison
(Dec. 2009 – Feb. 2016)

average difference 1.1×10^{-16}
statistical unc. down to 4.8×10^{-17}

Rb/Cs: search for annual variations



$$\frac{\Delta U(t)}{c^2} \simeq -\frac{GM_\odot}{c^2 a} \epsilon \cos[\Omega(t - t_{perihelion})]$$

$$d \ln\left(\frac{v_{Rb}}{v_{Cs}}\right) = C + (0.8 \pm 0.9) \times 10^{-16} \cos[\Omega_\oplus(t - t_{perihelion})]$$

$$c^2 \frac{d}{dU} \ln\left(\frac{v_{Rb}}{v_{Cs}}\right) = (-4.7 \pm 5.3) \times 10^{-7}$$

Differential redshift test

$$d\nu/\nu = (1 + \beta)dU/c^2$$

$$\beta(^{87}Rb) - \beta(^{133}Cs) = (-4.7 \pm 5.3) \times 10^{-7}$$

Variation of constants with gravity

$$c^2 \frac{d}{dU} \ln\left(\frac{g_{Rb}}{g_{Cs}} \alpha^{-0.49}\right) = (-4.7 \pm 5.3) \times 10^{-7}$$

$$\frac{d}{dt} \ln(\alpha^{-0.49} (m_q / \Lambda_{QCD})^{-0.021}) = (-4.7 \pm 5.3) \times 10^{-17} \text{ yr}^{-1}$$

Fundamental Physics with Ion Clocks: $^{201}\text{Hg}^+$ / $^{199}\text{Hg}^+$ Dual Isotope Clock

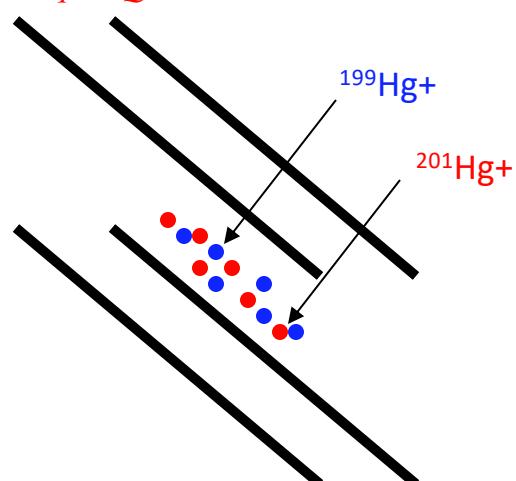
- HF clocks: depend on α , μ via $A \propto (m_e e^4/\hbar^2)[\alpha^2 F_{\text{rel}}(Z\alpha)](\mu m_e/m_p)$
 - some ambiguity
- Direct optical clock comparisons depend only on α
- $\mu \propto m_q/\Lambda_{QCD}$ *
- $B_{201} \approx -B_{199}$ **:

$$\frac{\partial}{\partial t} \ln \frac{f_{^{201}}}{f_{^{199}}} = \left[B_{^{201}} - B_{^{199}} \right] \frac{\partial}{\partial t} \ln \left(\frac{m_q}{\Lambda_{QCD}} \right)$$

- $B_{201} - B_{199} \approx 0.2$ - BIG!
- Would provide a stand-alone independent limit on m_q/Λ_{QCD} variation

Dual isotope clock will reduce systematic sensitivity in difference measurement:

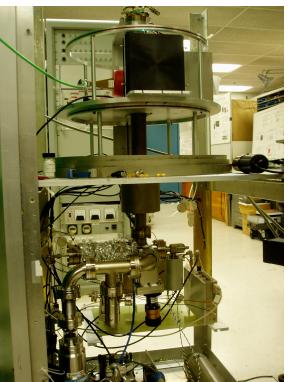
- Simultaneous measurement
- Measurement in same location



*V.V. Flambaum and A.F. Tedesco, Phys. Rev. C 73, 055501 (2006)

**S.N. Lea, to be published in the Eur. Phys. J ST.

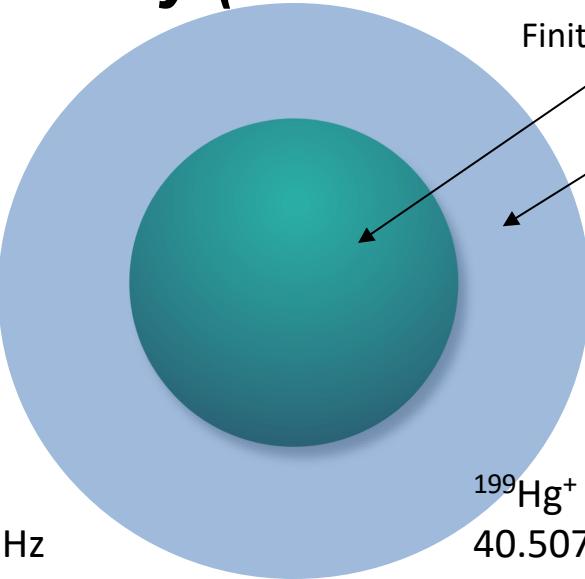
Fundamental Physics with Ion Clocks: Hyperfine Anomaly (Bohr-Weisskopf Effect*)



E.A. Burt, JPL

$^{201}\text{Hg}^+$ HF clock:
29.9543658213(17) GHz

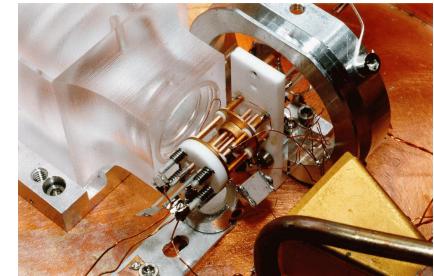
(E.A. Burt, et al., PRA 79, 062506 (2009))



Finite size nucleus

S electron:

Finite probability to be at nucleus



Courtesy J. Bergquist, NIST

$^{199}\text{Hg}^+$ HF clock:
40.50734799684159(41) GHz

(D.J. Berkeland, et al., PRL 80, 2089 (1998))

Point nucleus:

$$\frac{\Delta f_1}{\Delta f_2} = \left(\frac{\mu_{I1}/I_1}{\mu_{I2}/I_2} \right) \frac{F_1}{F_2}$$

Finite nucleus:

$$\frac{\Delta f_1}{\Delta f_2} = (1 + \Delta) \left(\frac{\mu_{I1}/I_1}{\mu_{I2}/I_2} \right) \frac{F_1}{F_2}$$

HF anomaly

*A. Bohr and V.F. Weisskopf, PR 77, 94 (1950)

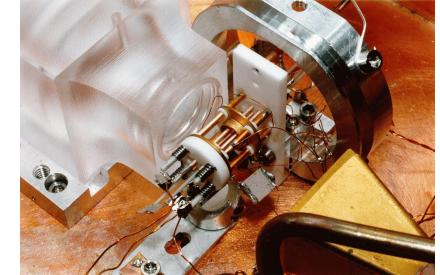
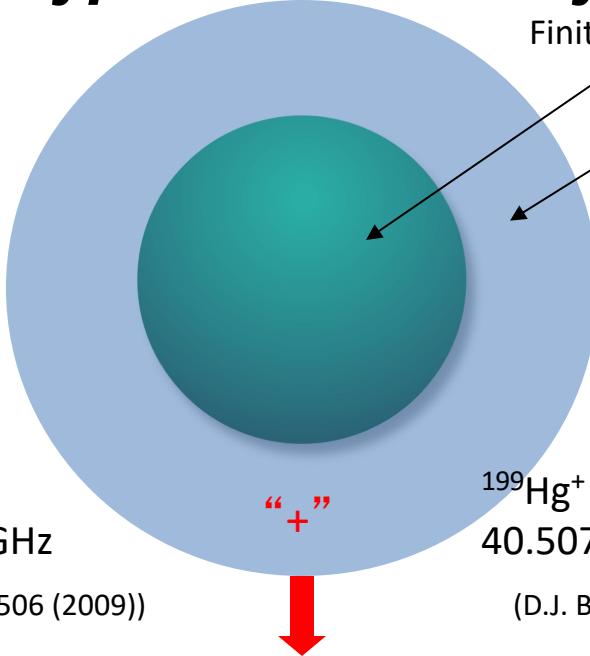
Fundamental Physics: The Hyperfine Anomaly



E.A. Burt, JPL

$^{201}\text{Hg}^+$ HF clock:
29.9543658213(17) GHz

(E.A. Burt, et al., PRA 79, 062506 (2009))



Courtesy J. Bergquist, NIST

$^{199}\text{Hg}^+$ HF clock:
40.50734799684159(41) GHz

(D.J. Berkeland, et al., PRL 80, 2089 (1998))

$$\frac{\Delta f_1}{\Delta f_2} = (1 + \Delta) \left(\frac{\mu_{I1}/I_1}{\mu_{I2}/I_2} \right) \frac{F_1}{F_2} \quad \frac{f_{201}}{f_{199}} = -0.739479805577(3)$$

$$\Delta(S_{1/2}, {}^{199}\text{Hg}^+, {}^{201}\text{Hg}^+) = -0.0016257(5)$$

E.A. Burt, et al., PRA 79, 062506 (2009)

Previous values Hg: -0.001627(19), (Reimann and McDermott, PRC 7, 2065 (1973))

Hg^+ : -.0034 to +0.0056 (Grandinetti, et al., (1986))

- Value now limited by knowledge of μ_l
- Agrees with neutral value: valence screening has minimal effect

Selected Textbook References

(see specific slides for journal references)

Atomic Physics

- “Elementary Atomic Structure”, 2nd Edition, G.K. Woodgate, Oxford (1986)
- “Atomic Physics”, C.J. Foot, Oxford (2005)

Trapping and Cooling of Atoms

- “Laser Cooling and Trapping”, H.J. Metcalf and P. van der Straten, Springer (1999)

Atomic Clocks

- “The Quantum Physics of Atomic Frequency Standards”, J. Vanier and C. Audoin, CRC Press
- “The Quantum Physics of Atomic Frequency Standards – Recent Developments”, J. Vanier and C. Tomescu, CRC Press (2016)

Clock Characterization

- “Characterization of Clocks and Oscillators”, NIST Technical Note 1337, D.B. Sullivan et al.