Emmy Nöther

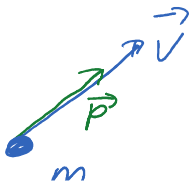
Erhaltungssätze $\hat{=}$ Symmetrie

Energie-erhaltung $\hat{=}$ Zeit symmetrie

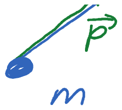
Impuls-erhaltung $\hat{=}$ Orts-symmetrie

Impulserhaltung (8.1.)

$$\vec{p} = m \cdot \vec{v}$$



$p = m \cdot v$



$[p] = 1 \frac{kg\,m}{s}$

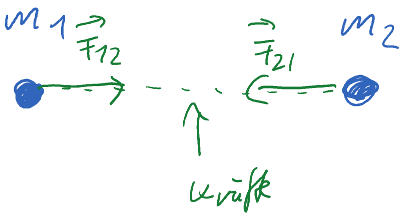
2. Newton'sches axiom

Beschleunigung

$\vec{a} = \frac{d}{dt}\vec{v}$

$\vec{F} = m \cdot \vec{a}$ Masse = const.

$$\Rightarrow \boxed{\vec{F} = \dot{\vec{p}} = \frac{d}{dt}\vec{p} = \frac{d}{dt}(m \cdot \vec{v})}$$



$\vec{F}_{12} = \frac{d}{dt}\vec{p}_1$ $\qquad$ $\vec{F}_{21} = \frac{d}{dt}\vec{p}_2$

actio = reactio $\qquad \vec{F}_{12} = -\vec{F}_{21} \quad \hat{=} \quad \underline{\vec{F}_{12} + \vec{F}_{21}} = 0$

$$\underline{\vec{F}_{12} + \vec{F}_{21}} = \frac{d}{dt}\vec{p}_1 + \frac{d}{dt}\vec{p}_2 = \frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$$

zeitl. Änderung des Gesamtimpulses $= 0$

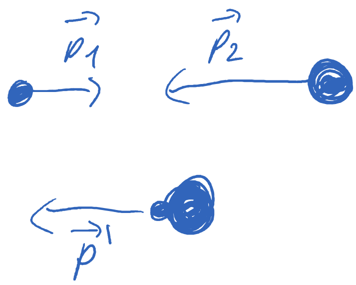
$\vec{p}_1 + \vec{p}_2 =$ konstant $\qquad$ Impulserhaltung

Stoßprozesse

elastische

inelastische Stöße ("Knete" $\rightarrow$ Verformungsarbeit)

Impulserhaltung



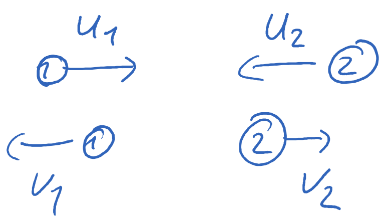
$$\vec{p}_1 + \vec{p}_2 = \vec{p}\,'$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\cdot\vec{v}\,'$$

elastische Stoß 1D

vorher: Geschw. u

nachher: v



$$\text{I} \quad \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

E-Erhaltung

$E_{kin} = \frac{1}{2}mv^2$

$$\text{II} \quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$u_1, v_2 > 0$ n. rechts

$u_2, v_1 < 0$ n. links

$$\text{I}\cdot 2, \quad m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$$

$$\text{II} \quad m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

elastisch $\rightarrow$ E-Erhaltung für E_{kin}

p-Erhaltung

aus I: $m_1(u_1-v_1)(u_1+v_1) = m_2(v_2-u_2)(v_2+u_2)$

$a^2 - b^2 = (a+b)(a-b)$ 3. bin. Formel

aus II $m_1(u_1-v_1) = m_2(v_2-u_2)$

$$\frac{\text{I}}{\text{II}} \quad \frac{m_1(u_1-v_1)(u_1+v_1)}{} = \frac{m_2(v_2-u_2)(v_2+u_2)}{}$$

$$\mathrm{I}/\mathrm{II} \quad \underbrace{\frac{m_1 (u_1 - v_1)}{m_1 (u_1 - v_1)}}_{=1}(u_1 + v_1) = \underbrace{\frac{m_2 (v_2 - u_2)}{m_2 (v_2 - u_2)}}_{=1}(v_2 + u_2)$$

$$u_1 + v_1 = v_2 + u_2$$

$$\Rightarrow \quad \underline{v_2 = u_1 + v_1 - u_2}$$

einsetzen in I

$$m_1 (u_1 - v_1)\cancel{(u_1 + v_1)} = m_2 (u_1 + v_1 - 2u_2)\cancel{(u_1 + v_1)}$$

$$m_1 (u_1 - v_1) = m_2 (u_1 + v_1 - 2u_2)$$

$$m_1 u_1 - m_1 v_1 = m_2 u_1 + m_2 v_1 - 2 m_2 u_2$$

$$-m_1 v_1 - m_2 v_1 = m_2 u_1 - 2 m_2 u_2 - m_1 u_1 \quad |\cdot(-1)$$

$$m_1 v_1 + m_2 v_1 = m_1 u_1 - m_2 u_1 + 2 m_2 u_2$$

$$v_1 (m_1 + m_2) = 2 m_1 u_1 + 2 m_2 u_2 - m_2 u_1 - m_1 u_1$$

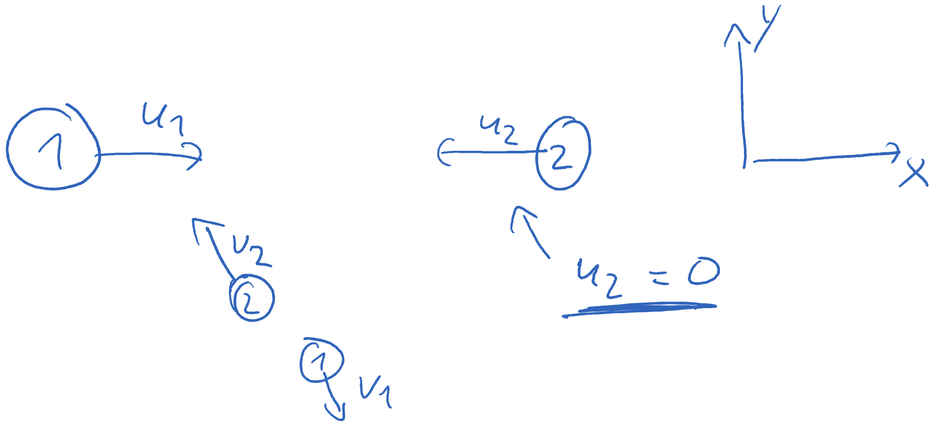
$$v_1 = 2\,\frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} - u_1$$

entsprechend $\quad v_2 = \underbrace{2\,\frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}}_{\text{Geschwindigkeit des Massenschwerpunkts (CMS Center of Mass System)}} - u_2$

Elastischer Stoß in 2D

vor

nachher



p-Erhaltung für jede Raumkomponente separat!

E-Erhaltung $\quad m_1 u_1^2 = m_1 v_1^2 + m_2 v_2^2$

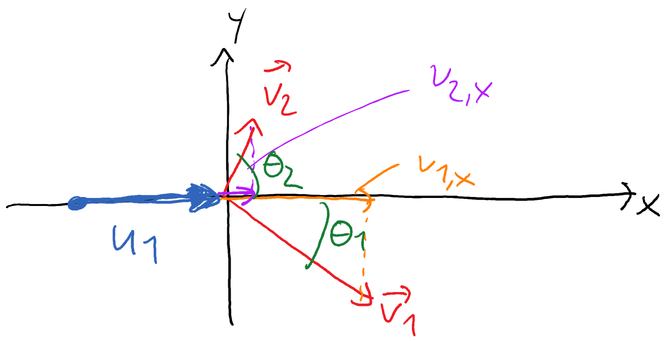
(u_2 sei $= 0$)

(Kugel 2 ruht vor dem Stoß)

p_x: p-Erhaltung $\quad x: \quad m_1 u_1 = m_1 v_{1,x} + m_2 v_{2,x}$

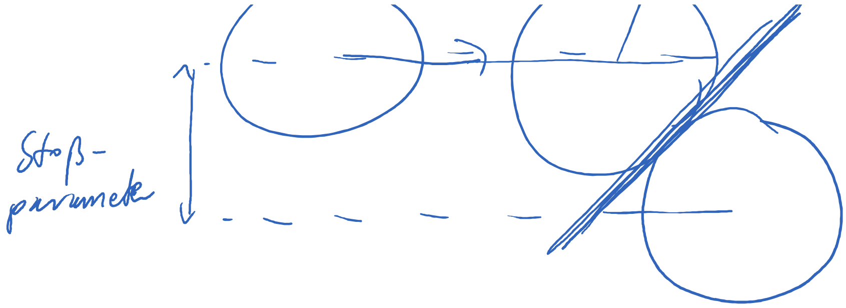
p_y: $\quad y: \quad 0 = m_1 v_{1,y} + m_2 v_{2,y}$

Polarkoordinaten



p_x: $\quad m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$

p_y: $\quad 0 = m_1 v_1 \sin\theta_1 + m_2 v_2 \sin\theta_2$



wenn 1 winkel oder v_1 oder v_2 bekannt

→ gelöst

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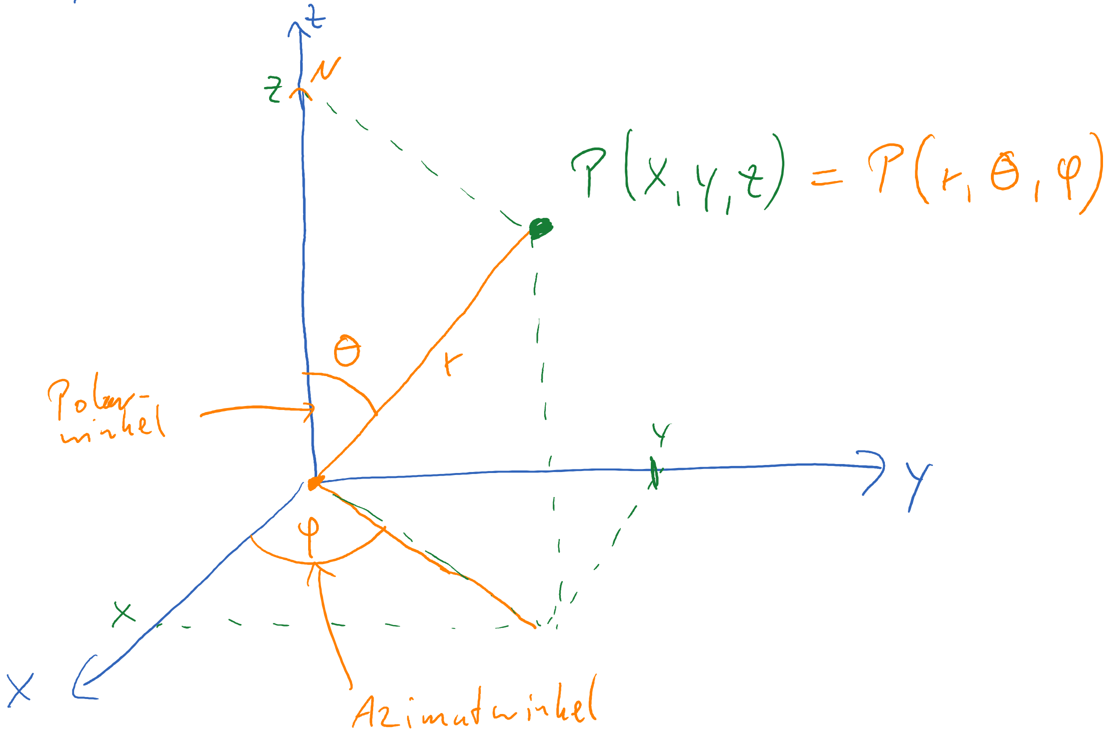
zu Blatt 2 Aufg. 1c :

Azimut- & Polarwinkel



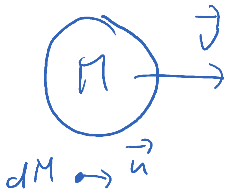
Impulserhaltung mit veränderlicher Masse

Rakete:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m \cdot \vec{v})$$

System Masse M Geschwindigkeit $\vec{v}$

Massenzuwachs dM " $\vec{u}$



$$\vec{p}_{vorher} = M\vec{v} + dM\cdot\vec{u}$$

$$\vec{p}_{nachher} = (M + dM)(\vec{v} + d\vec{v})$$

$$= \underbrace{\underline{M\cdot\vec{v}}}_{\text{groß, groß}} + \underbrace{dM}_{\text{klein}}\cdot\underbrace{\vec{v}}_{\text{groß}} + \underbrace{M}_{\text{groß}}\cdot\underbrace{d\vec{v}}_{\text{klein}} + \underbrace{dM\cdot d\vec{v}}_{\text{klein}\times\text{klein} = (\text{klein})^2 = \text{vernachlässigen}}$$

Näherung: erste 3 Terme

$$d\vec{p} = \vec{p}_{nach} - \vec{p}_{vorher} = dM\cdot\vec{v} + M\,d\vec{v} - dM\cdot\vec{u}$$

$$= M\cdot d\vec{v} - dM\,\vec{v}_{rel}$$

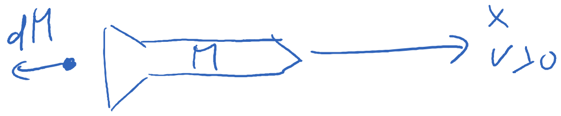
$$\vec{v}_{rel} = \vec{u} - \vec{v}$$

zeitl. Impulsänderung

$$\frac{d\vec{p}}{dt} = M\cdot\frac{d\vec{v}}{dt} - \frac{dM}{dt}\cdot\vec{v}_{rel} = \vec{F}_{ext}$$

$$M\cdot\vec{a} = \vec{F}_{ext} + \frac{dM}{dt}\cdot\vec{v}_{rel}$$

Rakete



$v_{rel} < 0$

$$\frac{dM}{dt} = \dot{M} < 0$$

Gesamtmasse der Rakete + Treibstoff nimmt ab

$$M(t) = M_0 + \dot{M} \cdot t$$

leerer Raum, keine Kraft $\rightarrow$ $\vec{F}_{ext} = 0$

$$\left(M_0 + \dot{M} \cdot t\right) \frac{d\vec{v}}{dt} = \dot{M} \cdot v_{rel}$$

konst

Separation der Variablen

$$\frac{d\vec{v}}{dt} = \dot{M} \cdot v_{rel} \cdot \frac{1}{M_0 + \dot{M} \cdot t}$$

$$\int d\vec{v} = \dot{M} \cdot v_{rel} \cdot \int \frac{1}{M_0 + \dot{M} \cdot t} \cdot dt$$

zeitlich konstant
$\Rightarrow$ vor's Integral

$$v(t) = v_{rel} \int \frac{1}{\frac{M_0}{\dot{M}} + t} \, dt$$

$$= v_{rel} \ln\left(\frac{M_0}{\dot{M}} + t\right) + C$$

$$= v_{rel} \ln\left(\frac{m_0}{\dot{m}} + t\right) + C$$

$$= v_{rel} \ln\left(\frac{m_0 + \dot{m} t}{\dot{m}}\right) + C$$

$$= v_{rel} \ln\left(\frac{m(t)}{\dot{m}}\right) + C$$

Integrationskonstante aus Anfangsbed

z.B. $v = 0$ für $t = 0$

$$C = - v_{rel} \ln \frac{m_0}{\dot{m}}$$

$$v(t) = v_{rel} \cdot \ln \frac{m(t)}{m_0}$$