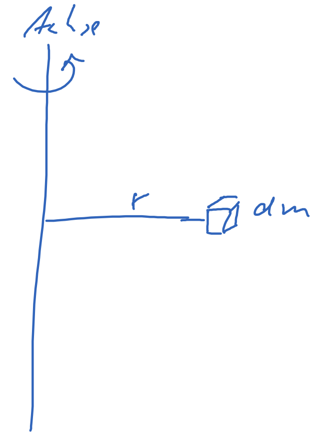
Der Trägheitstensor

Rufa 158 f



Zahl $J = \sum dm\, r^2$

$= \iiint r^2 \rho(\vec{r})\, dx\, dy\, dz$

↓

Tensor $\tilde{J} = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}$ Tensor

S. 159

Drehimpuls $\vec{L} = J \cdot \vec{\omega}$

$\vec{L}$ ↓ Vektor; $\vec{\omega}$ ↓ Vektor

J = Zahl

$\begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix} = \begin{pmatrix} J \cdot \omega_x \\ J \cdot \omega_y \\ J \cdot \omega_z \end{pmatrix}$

$\vec{L} = \tilde{J} \cdot \vec{\omega}$

$\tilde{J}$ → Tensor.

$\tilde{J} = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}$

$$\begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} \cdot \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} J_{11} \cdot \omega_x + J_{12} \cdot \omega_y + J_{13} \cdot \omega_z \\ J_{21} \cdot \omega_x + J_{22} \cdot \omega_y + J_{23} \cdot \omega_z \\ J_{31} \cdot \omega_x + J_{32} \cdot \omega_y + J_{33} \cdot \omega_z \end{pmatrix}$$

L_x wg ω_y

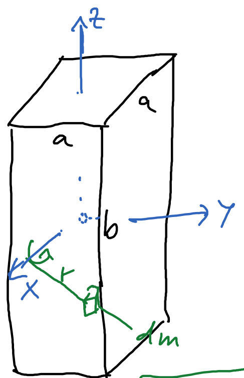
$$\begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}$$

← L_y

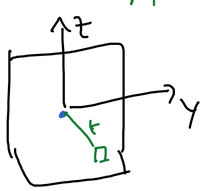
wg. ω_z

L_z

Roth, Stahl S. 281



$r = \sqrt{y^2 + z^2}$



$dm = \rho\, dV$

homogen

$$J_{11} = I_{xx} = \int_V (y^2+z^2)\,dm = \rho \int_{-b/2}^{b/2}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2} (y^2+z^2)\,dx\,dy\,dz$$

$$= \int_{-b/2}^{b/2}\int_{-a/2}^{a/2} \left[\frac{1}{3}y^3 + z^2 y\right]_{-a/2}^{+a/2} dx\,dz$$

$$= \int_{-b/2}^{b/2}\int_{-a/2}^{a/2} \frac{1}{12}a^3 + z^2 a \quad dx\,dz =$$

$$= \int_{-b/2}^{b/2} \left[\left(\frac{1}{12}a^3 + z^2 a\right)x\right]_{-a/2}^{a/2} dz =$$

$$= \int_{-b/2}^{b/2} \frac{1}{12}a^4 + z^2 a^2 \quad dz =$$

$$= \left[\frac{1}{12}a^4 z + \frac{1}{3}z^3 a^2\right]_{-b/2}^{b/2} =$$

$$= \boxed{\frac{1}{12}a^4 b + \frac{1}{12}b^3 a^2}$$

$$I_{xy} = J_{12} = -\int_V xy\,dm = -\rho \int_{-b/2}^{b/2}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2} xy\,dx\,dy\,dz$$

$-\frac{a}{2} \quad -\frac{a}{2} \quad -\frac{a}{2}$

$$= \iint \left[\frac{1}{2} x^2 y\right]_{-\frac{a}{2}}^{a/2} dy\, dz = \underline{\underline{0}}$$

$$J = \frac{1}{12} a^2 b \begin{pmatrix} a^2+b^2 & 0 & 0 \\ 0 & a^2+b^2 & 0 \\ 0 & 0 & 2a^2 \end{pmatrix}$$