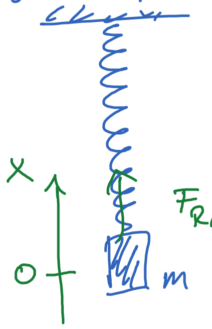
Freitag, 27. Mai 2022 10:24

DGL bei der gedämpften Schwingung

ungedämpfte Schwingung



$$F = m \cdot a = m \cdot \dot{v} = m \cdot \ddot{x}$$

$F_{RK} = -D \cdot x$

$$-D \cdot x = m \cdot \ddot{x} \qquad (7.4)$$

$$\ddot{x} + \frac{D}{m} x = 0$$

$$x = x(t) = \text{func}(t)$$

Ansatz z.B.: $x(t) = x_0 \cdot \cos(\omega_0 t)$ oder sin

$$\dot{x} = -x_0 \sin(\omega_0 t) \cdot \omega_0$$

$$\ddot{x} = -x_0 \omega_0^2 \cos(\omega_0 t)$$

$$-x_0 \omega_0^2 \cos \omega_0 t + \frac{D}{m} x_0 \cos \omega_0 t = 0$$

$$-\omega_0^2 + D/m = 0$$

$$\omega_0 = \sqrt{D/m}$$

(V) $m = 100g$ $\quad T_{10} = 11.7s$ $\quad$ 10 Oszillationen

$T = 1.2s$ $\quad \omega_0^{100g} = \frac{2\pi}{T}$

$m = 400g$ $\quad T_{10} = 21.4s$

$T = 2.1s$ $\quad \omega_0^{400g} = \frac{2\pi}{T} \approx \frac{1}{2} \omega_0^{100g}$

weil Massenverh. $= 1:4$ $+ \sqrt[4]{\;}^{4} \rightarrow 1:2$

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gedämpfte Schwingung

$F_{Reibung} = k \cdot v$ (Stokes'sche Reibung)

$\hookrightarrow \; v = \dot{x}$

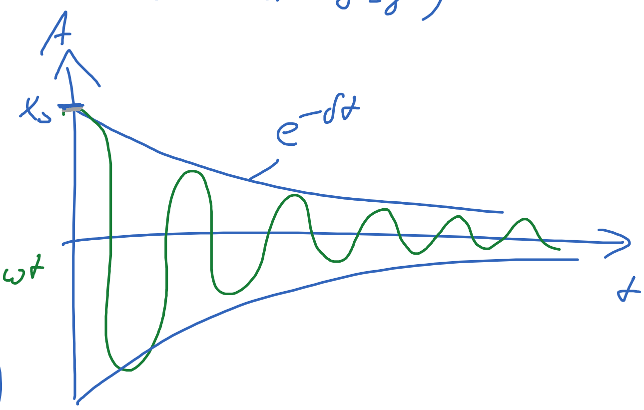
Bewegungsgleichung

$$m\ddot{x} + k\dot{x} + Dx = 0$$

Erwartung: Amplitude nimmt ab

$$x_0 \longmapsto x_0 e^{-\delta \cdot t}$$

(bei uns $\delta = \gamma$)

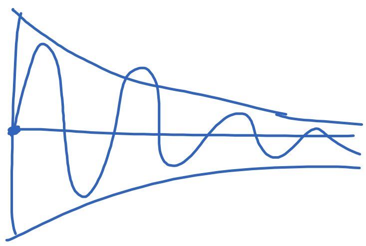


$$x(t) = x_0 e^{-\delta t} \cos \omega t$$

$$\dot{x}(t) = x_0(-\delta)e^{-\delta t} \cos \omega t + x_0 e^{-\delta t}(-\omega)\sin \omega t$$

$$= x_0 e^{-\delta t}\left(-\delta \cos \omega t - \omega \sin \omega t\right)$$

eher sin-mäßig



$$\ddot{x}(t) = x_0 \delta^2 e^{-\delta t} \cos \omega t + x_0 \delta e^{-\delta t} \omega \sin \omega t$$

$$+ x_0 \delta e^{-\delta t} \omega \sin \omega t - x_0 e^{-\delta t} \omega^2 \cos \omega t$$

$$= x_0 e^{-\delta t}\left[(\delta^2 - \omega^2)\cos \omega t + 2\delta\omega \sin \omega t\right)$$

Einsetzen: (in Bewegungsgl. geteilt durch m)

$$x_0 e^{-\delta t}\Big[(\delta^2 - \omega^2)\cos \omega t + 2\delta\omega \sin \omega t \qquad \ddot{x}$$

$$-\tfrac{k}{m}\delta \cos \omega t - \tfrac{k}{m}\omega \sin \omega t \qquad \dot{x}$$

$$+\tfrac{D}{m} \cos \omega t \Big] \overset{!}{=} 0 \qquad x$$

cos-Terme

sin-Terme:

$$2\delta\omega \sin \omega t - \tfrac{k}{m}\omega \sin \omega t \overset{!}{=} 0$$

$$\delta^2 - \omega^2 - 2\delta^2 + \omega_0^2 = 0$$

$$\omega_0^2 - \delta^2 - \omega^2 = 0$$

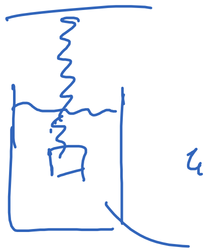
$$2\delta\omega \sin\omega t - \frac{k}{m}\omega \sin\omega t \stackrel{!}{=} 0$$

$$2\delta - \frac{k}{m} = 0$$

$$\boxed{\delta = \frac{k}{2m}}$$

Schwingungsfreq. $\omega = \sqrt{\omega_0^2 - \delta^2}$

Ⓥ



$T_5 = 9.4\,s$ 5 Schwingungen $T = \underline{1.8\,s}$

400g

Wasser zur Dämpfung

falsch gemessen

Fourier