Why not nuclear spin?

↳ YES!

So far $\vec{J} = \vec{L} + \vec{S}$ → fine structure

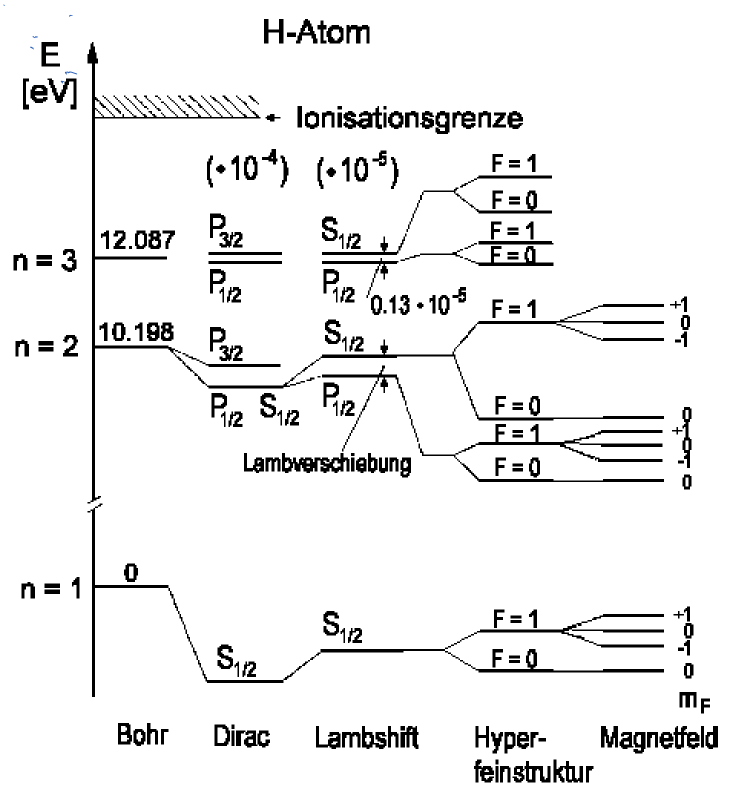
then $\vec{F} = \vec{J} + \vec{I}$ → hyperfine splitting

↳ nuclear spin

$\frac{1}{2}$ proton

1 deuteron

0 ^4He



Hydrogen

Equ. (3.14)

$$\begin{pmatrix} J & K \\ K & J \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \Delta E \begin{pmatrix} a \\ b \end{pmatrix}$$

$$(H_0 + H')\psi = E\psi \qquad (3.10)$$

$H_0 = H_1 + H_2$ $\quad$ $\frac{1}{r_{12}}$ e^-e^- Repulsion

H 1st e^- 2nd e^-

2 e^- in $Z=2$ field

pert. theory $\quad H'\psi = \Delta E \psi \qquad (3.12)$

$$\psi = a \cdot u_{1s}(1)\, u_{nl}(2) + b\, u_{1s}(2)\, u_{nl}(1) \qquad (3.13)$$

ψ into 3.12

multiply by $\boxed{u_{1s}^*(1)\, u_{nl}^*(2)}$ $\quad$ I

or $u_{1s}^*(2)\, u_{nl}^*(1)$ $\quad$ II $\quad$ } 2 coupled equ

and then $\iint d\vec{r}_1 d\vec{r}_2$ in (3.12)

$$\iint H' \left[a \cdot u_{1s}(1)\, u_{1s}^*(1)\, u_{nl}(2)\, u_{nl}^*(2) + b \cdot u_{1s}(2)\, u_{1s}^*(1)\, u_{nl}(1)\, u_{nl}^*(2) \right] d\vec{r}_1 d\vec{r}_2$$

$H' = \frac{e^2}{r_{12}}$ $\quad$ $J \cdot a$ $\quad$ $K \cdot b$

$$1 = \int d\vec{r}_1\, |u_{1s}(1)|^2 \cdot \int d\vec{r}_2\, |u_{nl}(2)|^2 = 1$$

$$= \Delta E \iint a\, u_{1s}(1)\, u_{1s}^*(1) \cdot u_{nl}(2)\, u_{nl}^*(2) + b\, u_{1s}(2)\, u_{1s}^*(1)\, u_{nl}(1)\, u_{nl}^*(2)\, d\vec{r}_1 d\vec{r}_2$$

$=0$ $\quad$ $=0$

$H'\psi = \Delta E\psi$ $\quad \to \frac{e^2}{r_{12}}$

$\Delta E \iint$ same without $\boxed{\frac{1}{r_{12}} = H'}$

$\int u_{1s}(2)\, u_{nl}^*(2)\, d\vec{r}_2 = 0$

because $nl \neq 1s$ "excited states"

$$J = \frac{1}{4\pi\varepsilon_0} \iint |u_{1s}(1)|^2 \frac{e^2}{r_{12}} |u_{nl}(2)|^2 \, d\vec{r}_1 \, d\vec{r}_2$$

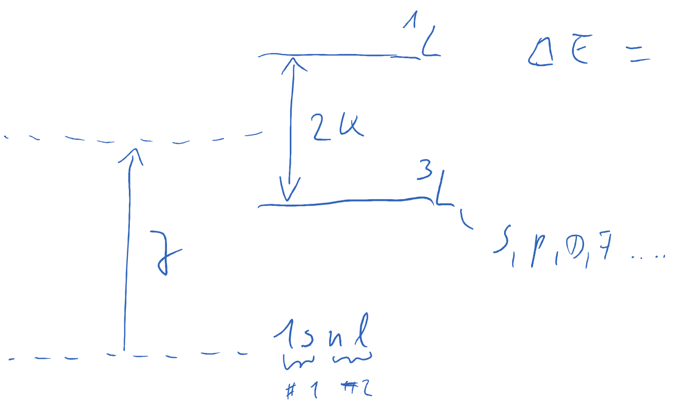
direct integral

$$\left.\begin{array}{lllll} \mathrm{I} \Rightarrow & a \cdot J & + \; b \cdot K & = & \Delta E \cdot a \\ \mathrm{II} \Rightarrow & a \cdot K & + \; b \cdot J & = & \Delta E \cdot b \end{array}\right\}$$

$$\begin{pmatrix} J & K \\ K & J \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \Delta E \begin{pmatrix} a \\ b \end{pmatrix}$$

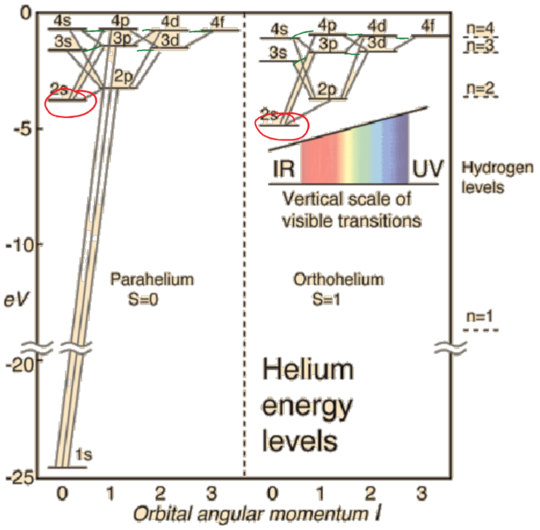
$$(3.17) \qquad \begin{vmatrix} J - \Delta E & K \\ K & J - \Delta E \end{vmatrix} = 0$$

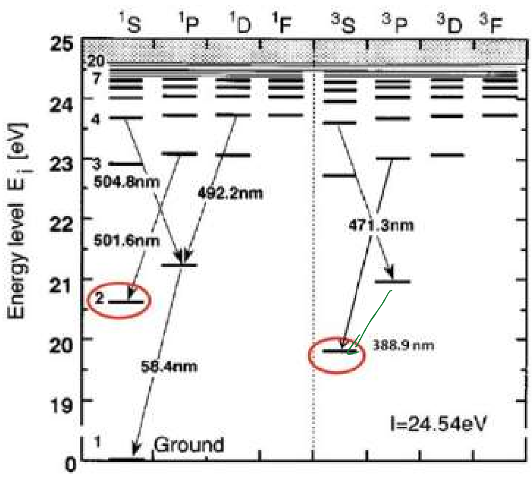
$$\Delta E = J \pm K$$

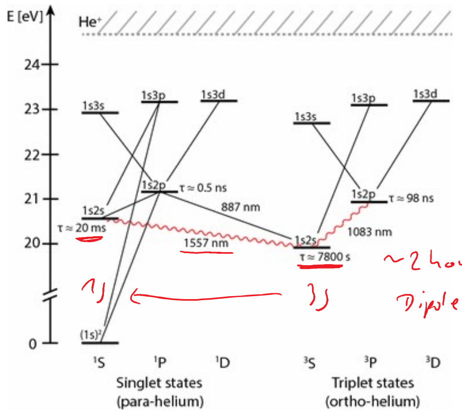


$$K = \frac{1}{4\pi\varepsilon_0} \iint u_{1s}^*(1) \, u_{nl}^*(2) \, \frac{e^2}{r_{12}} \, u_{1s}(2) \, u_{nl}^*(1) \, d\vec{r}_1 \, d\vec{r}_2$$

exchange







~2 hours

no Dipole transition allowed

long lifetimes → narrow lines

attractive for spectroscopy

Dipole-forbidden, but "a little bit" allowed by higher order radius