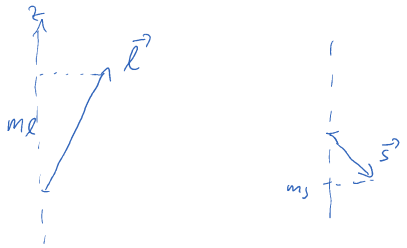
Clebsch-Gordan coefficients

example spin-spin or spin-orbit interaction

$|l, m_l\rangle \qquad |s, m_s\rangle$



$\vec{j} = \vec{l} + \vec{s}$

$|l, m_l\rangle \otimes |s, m_s\rangle \equiv |l, m_l, s, m_s\rangle$

$n = 1 \dots \infty$

$l = 0 \dots n-1 \qquad m_l = -l \dots l$

$s = 1/2 \qquad m_s = -s \dots s$

$\rightarrow j = l \pm s$ but $j \geq 0 \qquad m_j = -j \dots j$

$$|l - s| \leq j \leq l + s$$

1. spin 2. spin

(1) $\hat{=}$ 2 s electrons in He

$s_1 = 1/2 \quad s_2 = 1/2$

$l = 1/2 \quad s = 1/2$

singlett, triplett

total spin $j = \frac{1}{2} \pm \frac{1}{2} = 0, 1$

(1a) $j = 0 \Rightarrow m_j = 0 \qquad m_j \in \{-j \dots j\}$

singlett

$$|j = 0, m_j = 0\rangle = \sum_{m_l + m_s = m_j} \underbrace{\langle m_l, m_s | j = 0, m_j = 0 \rangle}_{CG} |m_l, m_s\rangle$$

$$= \langle \uparrow\downarrow | 0,0 \rangle |\uparrow\downarrow\rangle + \langle \downarrow\uparrow | 0,0 \rangle |\downarrow\uparrow\rangle$$

$$= \frac{1}{\sqrt{2}} |\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}} |\downarrow\uparrow\rangle$$

1b: Triplett

$j = 1, \quad m_j = \{-1, 0, 1\}$

$$|j = 1, m_j = 1\rangle = \sum \langle m_l m_s | j = 1, m_j = 1 \rangle | m_l m_s \rangle$$

$m_j = m_{j_1} + m_{j_2}$

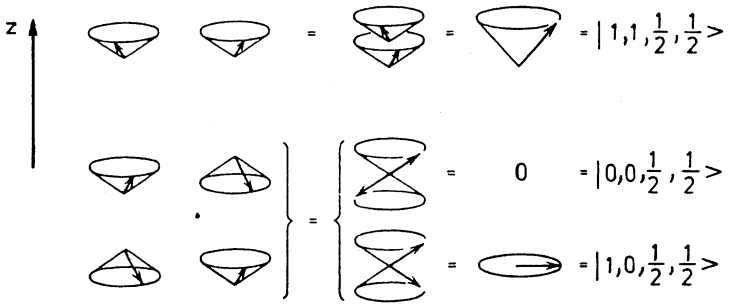
$= \dots \; 1 \,|\uparrow\uparrow\rangle$

$\tfrac{1}{2}\uparrow_{1/2} + \tfrac{1}{2}\uparrow_{1/2} = m_j = 1 \uparrow$

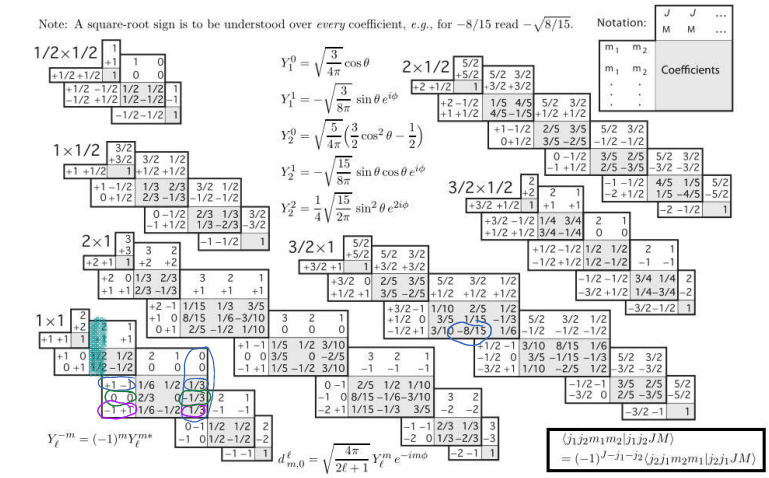
$j(j+1) \geq m_j$

$|j=1;\ m_j=-1\rangle \;\dots = 1\,|\downarrow\downarrow\rangle$

$|j=1,\ m_j=0\rangle = \frac{1}{\sqrt{2}}\,|\uparrow\downarrow\rangle \;(+)\; \frac{1}{\sqrt{2}}\,|\downarrow\uparrow\rangle$



44. Clebsch-Gordan Coefficients, Spherical Harmonics, and *d* Functions



(2) $j_1 = 1\,,\ j_2 = 1$

$j = 0, 1, 2 \qquad m_j = -2, -1, 0, 1, 2$

(2a) $|j=0\,,\ m_j=0\rangle = \sqrt{\tfrac{1}{3}}\,|m_{j1}=1,\ m_{j2}=-1\rangle$

$\to -\sqrt{\tfrac{1}{3}}\,|m_{j1}=0,\ m_{j2}=0\rangle$

$+\sqrt{\tfrac{1}{3}}\,|-1\,,\ 1\rangle$

(2b) $|j=1,\ m_j=1\rangle = \sqrt{\tfrac{1}{2}}\,|1,0\rangle + \sqrt{\tfrac{1}{2}}\,|0,1\rangle$ (m_{j1}, m_{j2})

Many-electron atoms

$\hat{H} = \sum_i \left(\frac{p_i^2}{} - \frac{Ze^2}{} \right) + \sum \frac{e^2}{} \to$

Many - electron atoms

$$\hat{H} = \sum_{i=1}^{Z} \left(\frac{p_i^2}{2m_e} - \frac{Ze^2}{4\pi\varepsilon_0 r_i} \right) + \sum_{i>j} \frac{e^2}{4\pi\varepsilon_0 |\vec{r}_i - \vec{r}_j|}$$

difficult

e - e repulsion

Pauli exclusion principle

↳ only $2e^-$ in each subshell

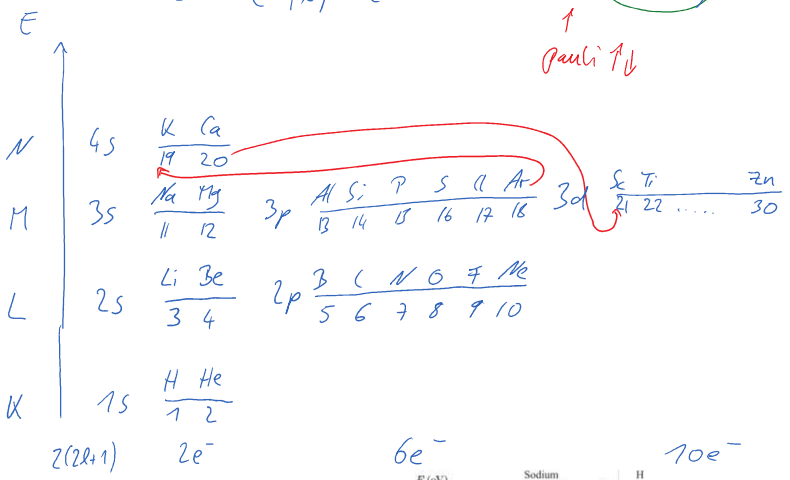
$n = 0 \ldots$

each n state has n values of ℓ: $\ell = 0, 1, \ldots, n-1$

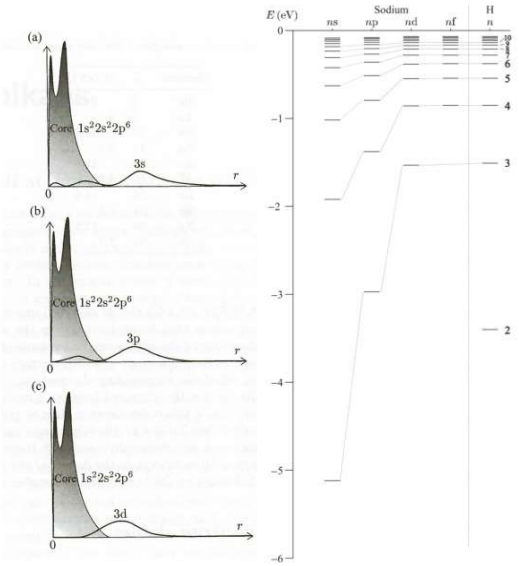
ℓ $(2\ell+1)$ m_ℓ: $m_\ell = -\ell, \ldots \ell$

each (n,ℓ) can hold $2 \times (2\ell+1)$

↑ Pauli ↑↓



Na



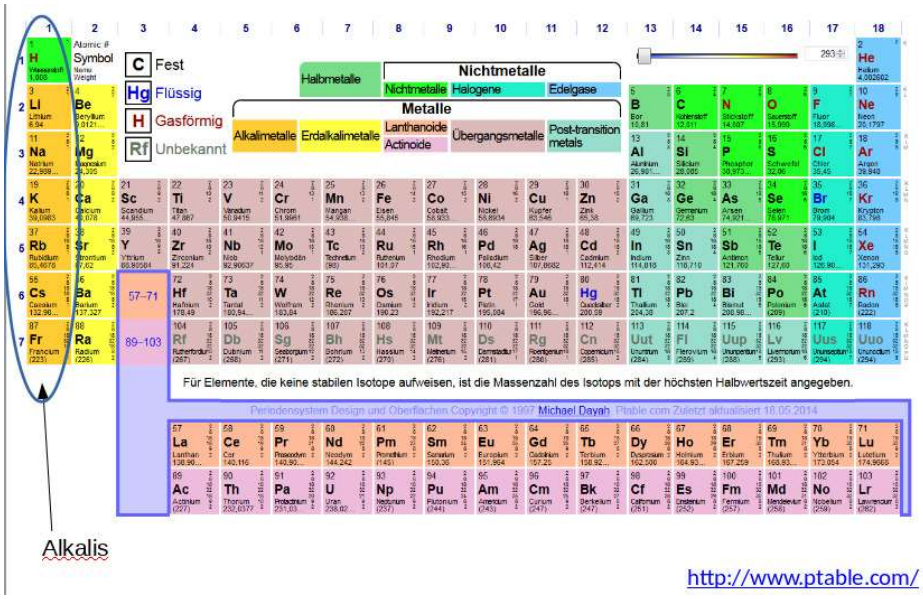
Foot, Atomic Physics

Foot, Atomic Physics

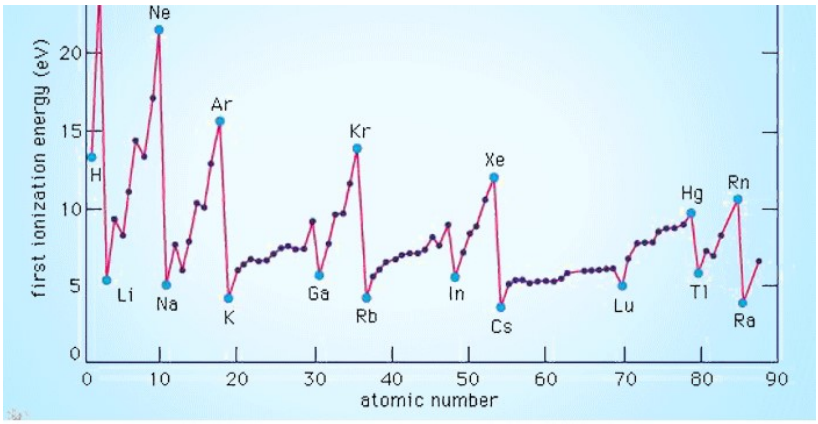
Fig. 4.2 The energies of the s, p, d and f configurations in sodium. The energy levels of hydrogen are marked on the right for comparison. The guidelines link configurations with the same n to show how the energies become closer to the hydrogenic values as l increases, i.e. the quantum defects decrease so that $\delta_l \simeq 0$ for f-electrons (and for the configurations with $l > 3$ that have not been drawn).

energetically favourable to occupy 4s shell before the 3d shell

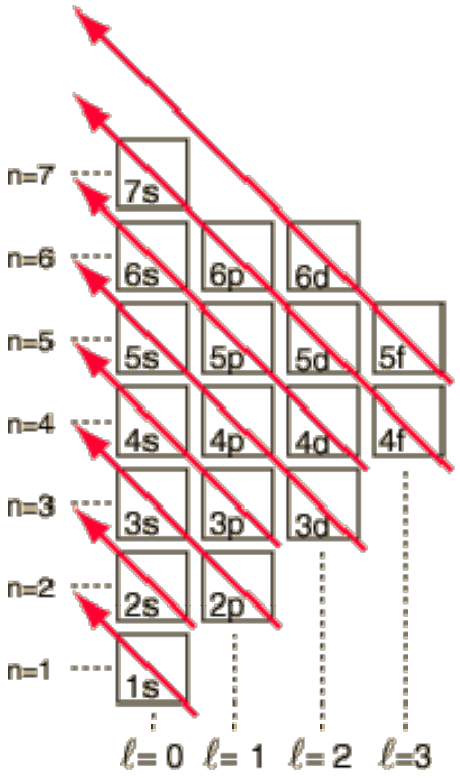
→ chemical properties



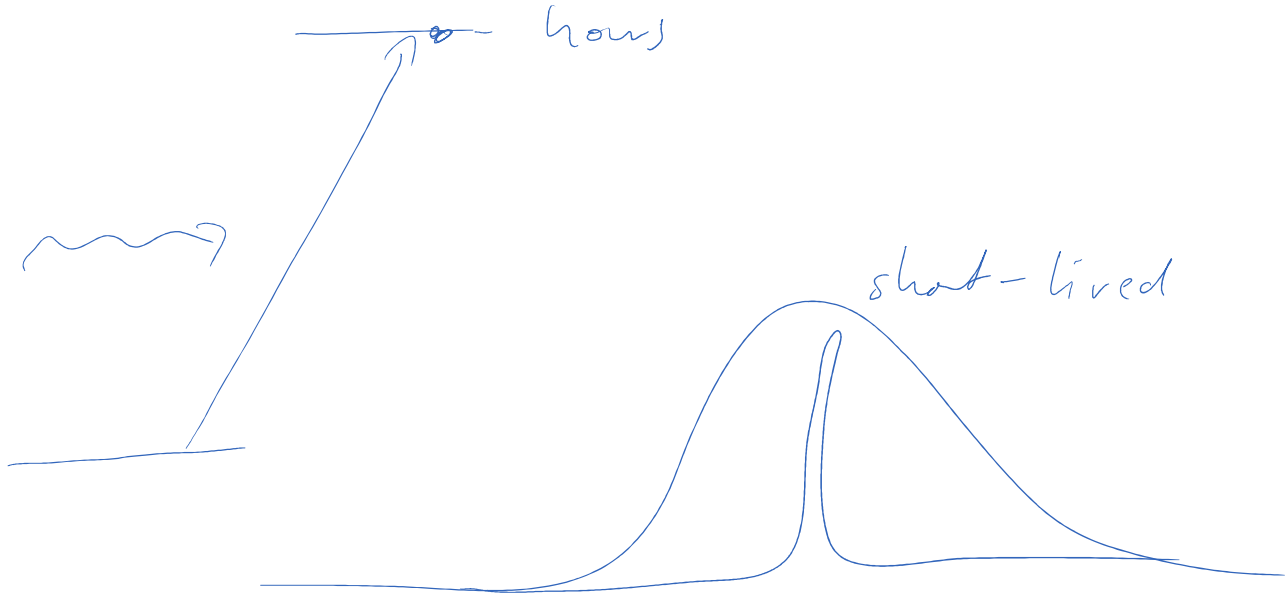
http://www.ptable.com/



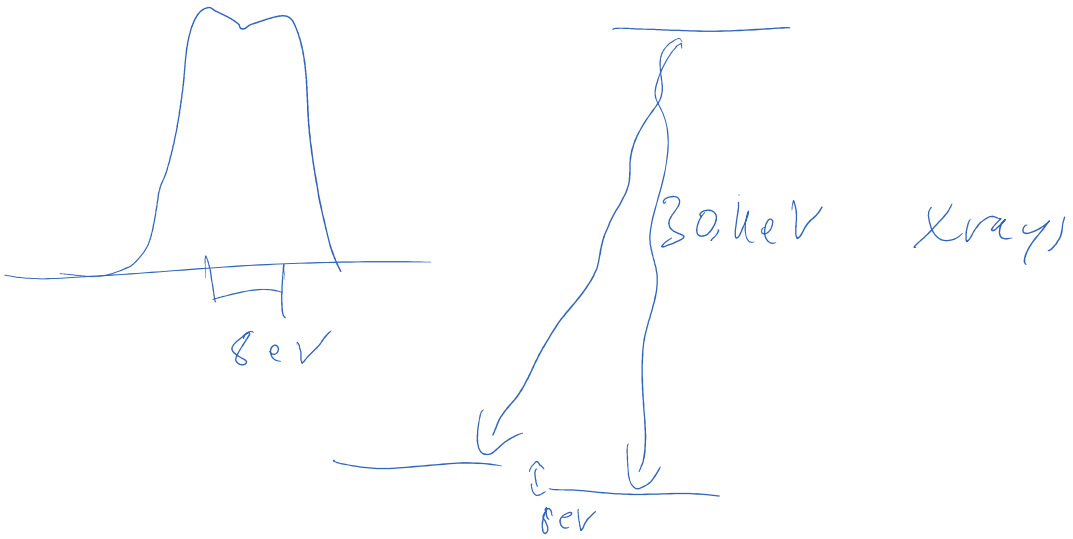
THE IONIZATION ENERGY OF THE GROUP 1 ELEMENTS



Lars van der Wense MPQ, LMU



^{229}Th $\Delta E \simeq 8\,eV$


8eV
30keV
Xrays
8eV