Zeeman effect

Donnerstag, 1. Dezember 2022 12:45

Hertel, Schulz
Atoms, Molecules & optical physics 1

usual atom with spin-orbit interaction

$$\hat{V}_{LS} = a \cdot \hat{\vec{L}} \cdot \hat{\vec{S}}$$

$\hookrightarrow$ s-o-interaction parameter

expectation value = fine structure splitting

$$V_{LS} = \langle \hat{V}_{LS} \rangle = \frac{a}{2} \left[J(J+1) - L(L+1) - S(S+1) \right]$$

see interval rule in Foot Fig. 5.6.

SI units $\hat{V}_{LS} = \frac{a \hat{\vec{L}} \hat{\vec{S}}}{\hbar^2}$

L, S, J orbital ang. mom., spin, tot. ang. mom

M_L, M_S, M_J = projections "magnetic quantum numbers"

Let's add a $\vec{B}$-field $\hat{=}$ z direction

interaction energy $\hat{V}_B$ depends

on magnetic moments $\vec{\mu}_L$, $\vec{\mu}_S$, $\vec{\mu}_J = \vec{\mu}_L + \vec{\mu}_S$

g factors $g_L = 1$ (orbit)

$g_s = 2$ (spin)

Bohr magneton $\mu_B = \frac{e\hbar}{2m_e}$

$$\hat{V}_B = -\hat{\vec{\mu}}_J \cdot \vec{B} = -\left(\hat{\vec{\mu}}_L + \hat{\vec{\mu}}_S\right)\vec{B}$$

$$= \frac{\mu_B}{\hbar}\left(\underset{\underset{1}{\uparrow}}{g_L} \cdot \hat{L}_z + \underset{\underset{2}{\uparrow}}{g_s} \cdot \hat{S}_z\right) \cdot B$$

$$= \mu_B \frac{\hat{J}_z + \hat{S}_z}{\hbar} B = -\hat{\mu}_{J_z} \cdot B$$

total $\hat{H} = \hat{H}_0 + \hat{V}_{LS} + \hat{V}_B$

$$= \underset{\substack{\uparrow \\ \text{unperturbed} \\ \text{"Coulomb atom"}}}{\hat{H}_0} + \underset{\text{S-O coupling}}{a\frac{\hat{\vec{J}}^2 - \hat{\vec{L}}^2 - \hat{\vec{S}}^2}{2\hbar^2}} + \underset{\text{magnetic interaction}}{\mu_B \frac{\hat{J}_z + \hat{S}_z}{\hbar} B}$$

$$H_0 |n L M_L S M_S\rangle = W_{nLS} |n L M_L S M_S\rangle$$

these are not good quantum numbers any more

M_J is still a good quantum number

B field		perturbation	basis	name
low	$B \ll \frac{a}{\mu_B}$	$\hat{V}_B \ll \hat{V}_{LS}$	$\|LSJM_J\rangle$	anomalous Zeeman effect
high	$\gg$	$\gg$	$\|LM_L S M_S\rangle$	Paschen Back effect

B fields in lab: 1-5 Tesla
30 Tesla SC magnets

$$\mu_B = 5.788 \cdot 10^{-5} \frac{eV}{T}$$

$$\langle V_B \rangle \approx 2 \cdot 10^{-3} eV = 16 cm^{-1}$$

$\underline{meV}$

electronic excitation $\hat{=}$ vis. light $\approx$ 3eV

FS splitting P state

H	$0.33 cm^{-1}$
Li	$0.36 cm^{-1}$
He	$\sim 1 cm^{-1}$

Zeeman shift in low fields

0^{th} order $\quad H = \hat{H}_0 + V_{LS}$

eigenstates $|J, M_J\rangle$

$2S+1$ substates with

$2J+1$ degenerate magnetic sublevels

$\hat{V}_B$ = 1st order perturbation

⋮

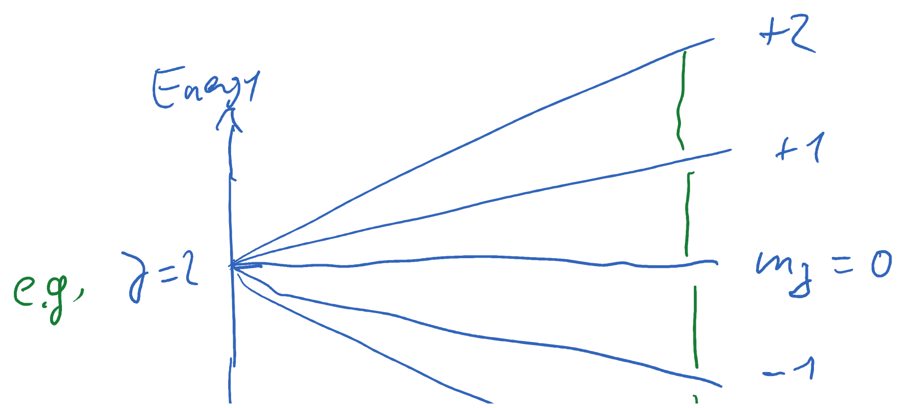
$$V_B = \underbrace{\left[1 + \frac{\langle J M_J | \hat{\vec{S}} \cdot \hat{\vec{J}} | J M_J \rangle}{J(J+1)\hbar^2}\right]} \mu_B M_J B$$

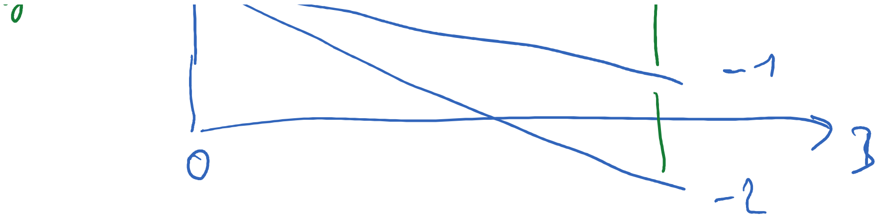
Landé g factor

$$g_J = \frac{3J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$$V_B = g_J \cdot \mu_B \cdot M_J \cdot B = -\underline{\mu_{J_z}} \cdot B$$

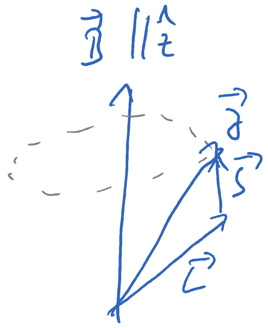
low fields: each J level splits into $2J+1$ equally spaced levels

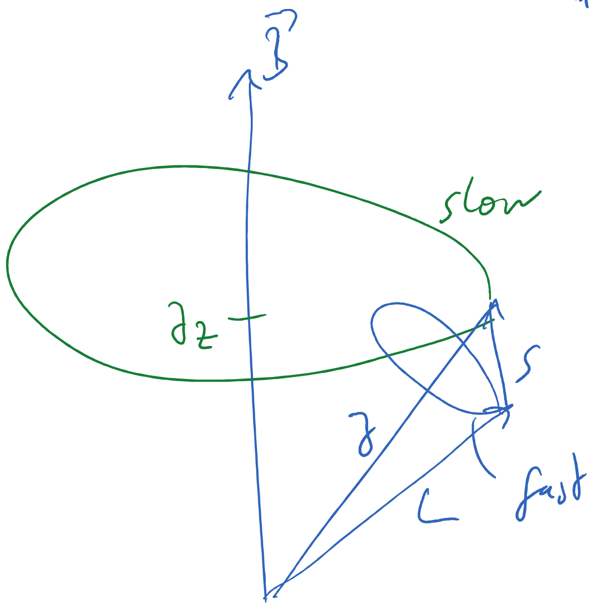




equally spaced

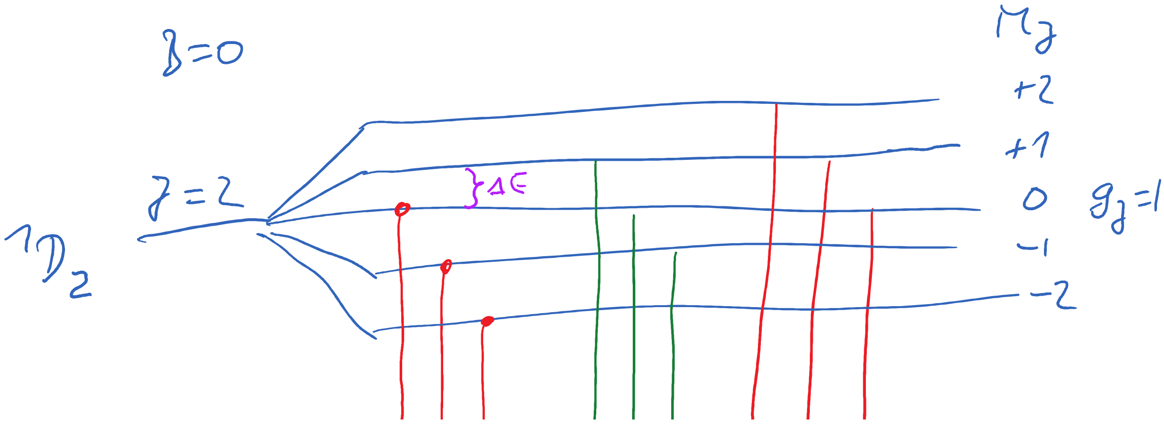
Vector model

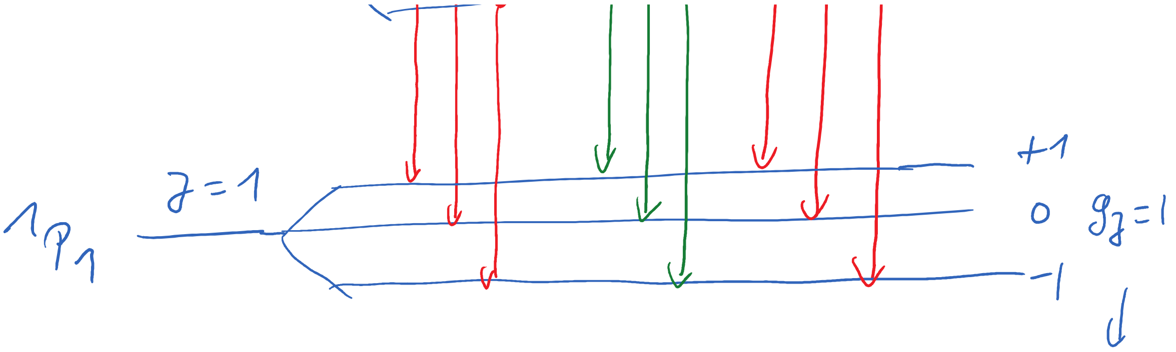




Examples:

- singlett states



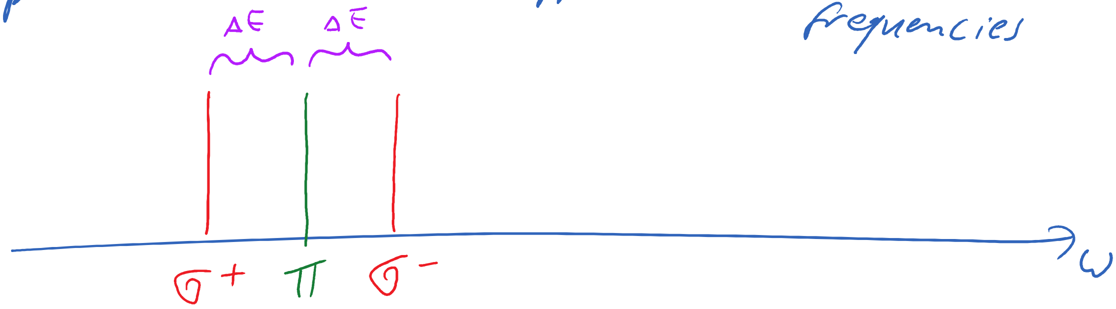


transition $\Delta M_J=+1$ $\quad$ $M_J=0$ $\quad$ $\Delta M_J=-1$ $\quad$ equal splitting

σ_+ $\quad$ π $\quad$ σ_- light

circular polarized light $\quad$ lin. pol. light

9 possible transitions appear at 3 different frequencies



normal zeeman effect
He singlet states

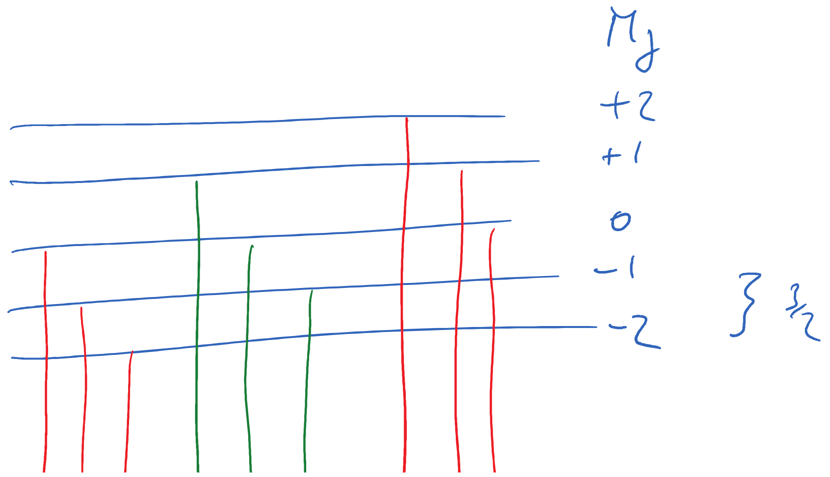
can NOT be observed along $\vec{B}$!!

triplett system

$^3P_2$

$J=2$

$g_J=3/2$

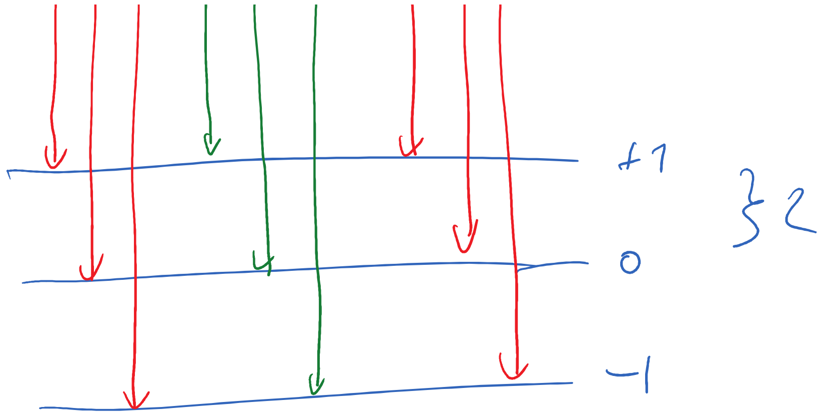


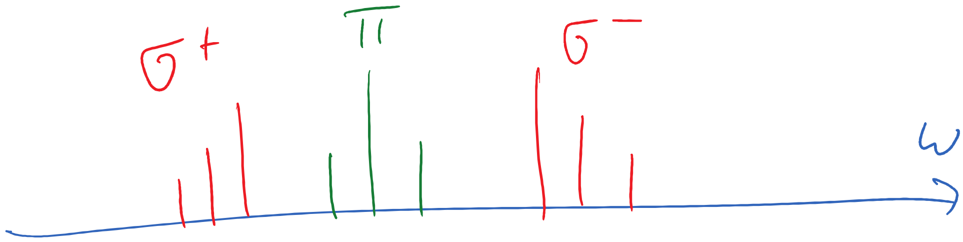
0 0

$^3S_1$

$J = 1$

$g_J = 2$





all 9 lines separated