OBE part 2

(7.34, 35)

$$\tilde{c}_1 = c_1 e^{-i\frac{\delta}{2}t} \qquad \delta = \omega - \omega_0$$

$$\tilde{c}_2 = c_2 e^{+i\frac{\delta}{2}t} \qquad \text{detuning}$$

density matrix

$$\rho = \begin{pmatrix} \underbrace{c_1 c_1^*}_{|c_1|^2} & c_1 c_2^* \\ c_1^* c_2 & \underbrace{c_2 c_2^*}_{|c_2|^2} \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$

coherences populations

population: probability to find atom in state $|1\rangle$ or $|2\rangle$

coherences: are related to the phase between the states

in particular, if $\rho_{12} = \rho_{21} = 0$

the phase between $|1\rangle$ and $|2\rangle$ is undefined

$\rightarrow$ statistical mixture of atoms

Example: $c_1 = c_2 = \frac{1}{\sqrt{2}}$

$$\rightarrow \rho = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

populations = 50:50

coherences give a well-defined phase between the states

$$\Psi(\vec{r},t) = c_1 e^{-i\omega_1 t} u_1(\vec{r}) + c_2 e^{i\varphi} e^{-i\omega_2 t} u_2(\vec{r})$$

time evolution of the elements of the density matrix gives the physics going on

$$\dot{\rho}_{11} = i\frac{\Omega_0}{2}\left(\tilde{\rho}_{21} - \tilde{\rho}_{12}\right) \qquad \Omega_0 \mathbin{/\!/} \Omega$$

$$\dot{\rho}_{22} =$$

Foot (7.42)

$$\dot{\rho}_{12} =$$

$$\dot{\rho}_{21} =$$

We can introduce incoherent processes (e.g. spontaneous decay $|2\rangle \to |1\rangle$):

$$\boxed{\begin{aligned}
\dot{\rho}_{11} &= \gamma \rho_{22} + i \frac{\Omega_0}{2} \left(\tilde{\rho}_{21} - \tilde{\rho}_{12} \right) \\
\dot{\rho}_{22} &= -\gamma \rho_{22} + i \frac{\Omega_0}{2} \left(\tilde{\rho}_{22} - \tilde{\rho}_{21} \right) \\
\dot{\tilde{\rho}}_{12} &= -\left(\frac{\gamma}{2} + i\delta \right) \tilde{\rho}_{12} + i \frac{\Omega_0}{2} \left(\rho_{22} - \rho_{11} \right) \\
\dot{\tilde{\rho}}_{21} &= -\left(\frac{\gamma}{2} - i\delta \right) \tilde{\rho}_{21} + i \frac{\Omega_0}{2} \left(\rho_{11} - \rho_{22} \right)
\end{aligned}}$$

The Bloch sphere

$$\rho_{11} + \rho_{22} = 1$$

u, v, w

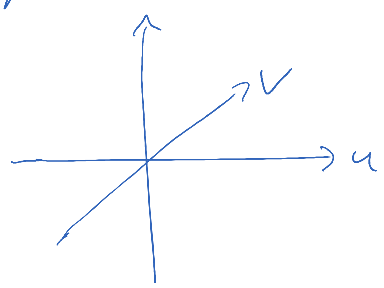
7.41, 45, 46 $\to$ (7.67)

$$\boxed{\begin{aligned}
\dot{u} &= \delta \cdot v - \frac{\Gamma}{2} u \\
\dot{v} &= -\delta u + \Omega w - \frac{\Gamma}{2} v \\
\dot{w} &= \Omega v - \Gamma (w - 1)
\end{aligned}}$$

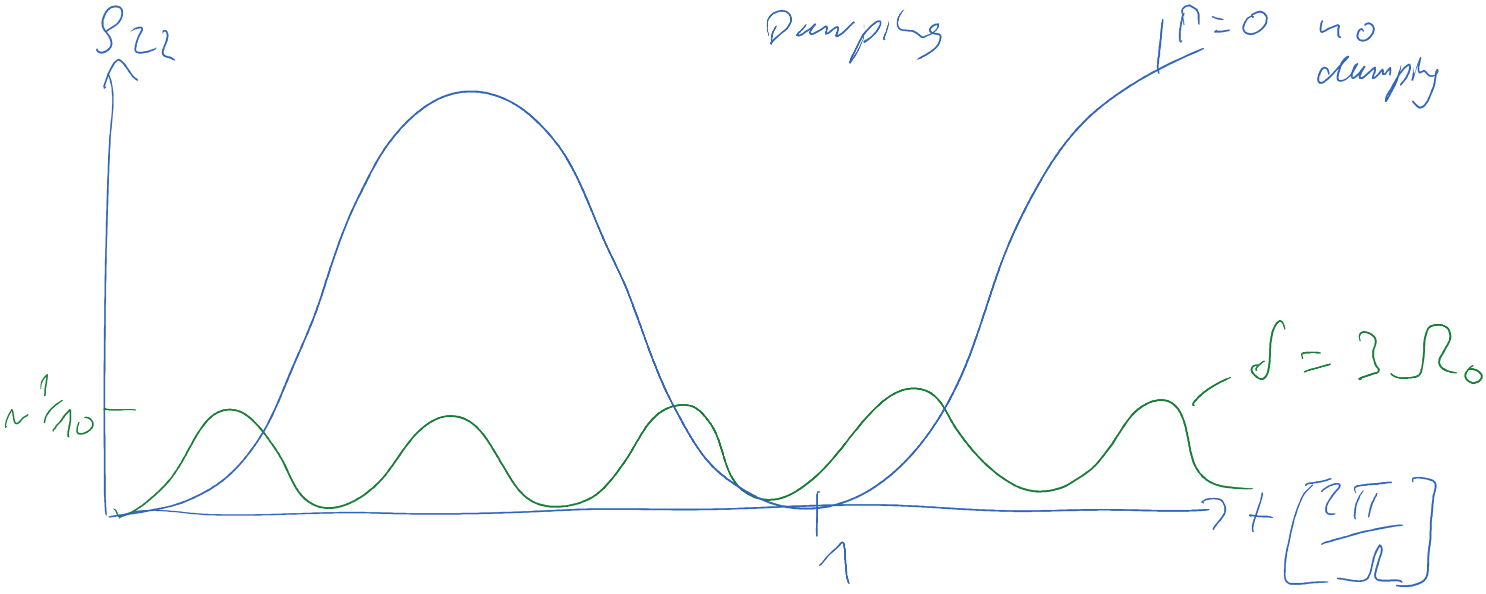
$\Gamma = \gamma$ spontaneous decay rate

$w = \rho_{11} - \rho_{22}$ inversion

Bloch sphere



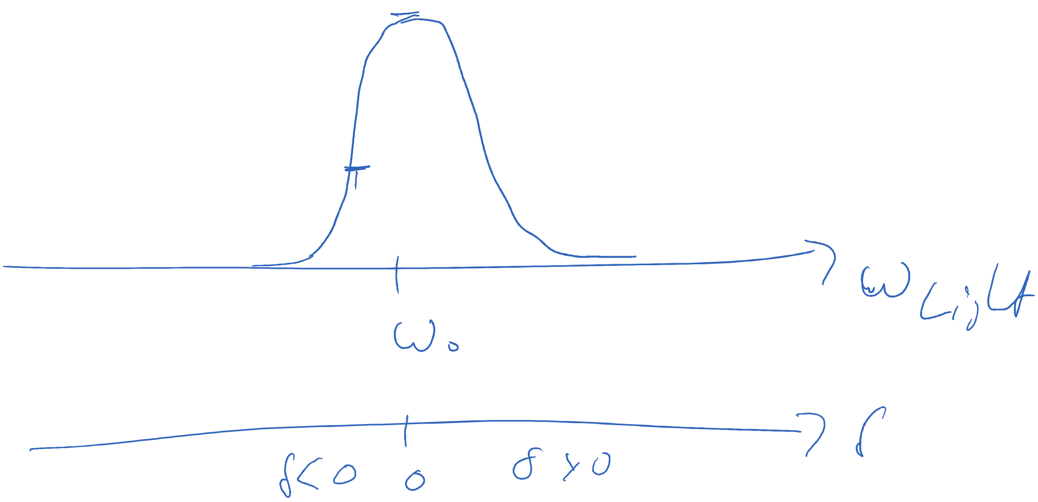
$$\begin{pmatrix} \dot u \\ \dot v \\ \dot w \end{pmatrix} = \begin{pmatrix} \Omega \\ 0 \\ -\delta \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} - \underbrace{\Gamma \begin{pmatrix} u/2 \\ v/2 \\ w+1 \end{pmatrix}}_{\text{Damping}}$$



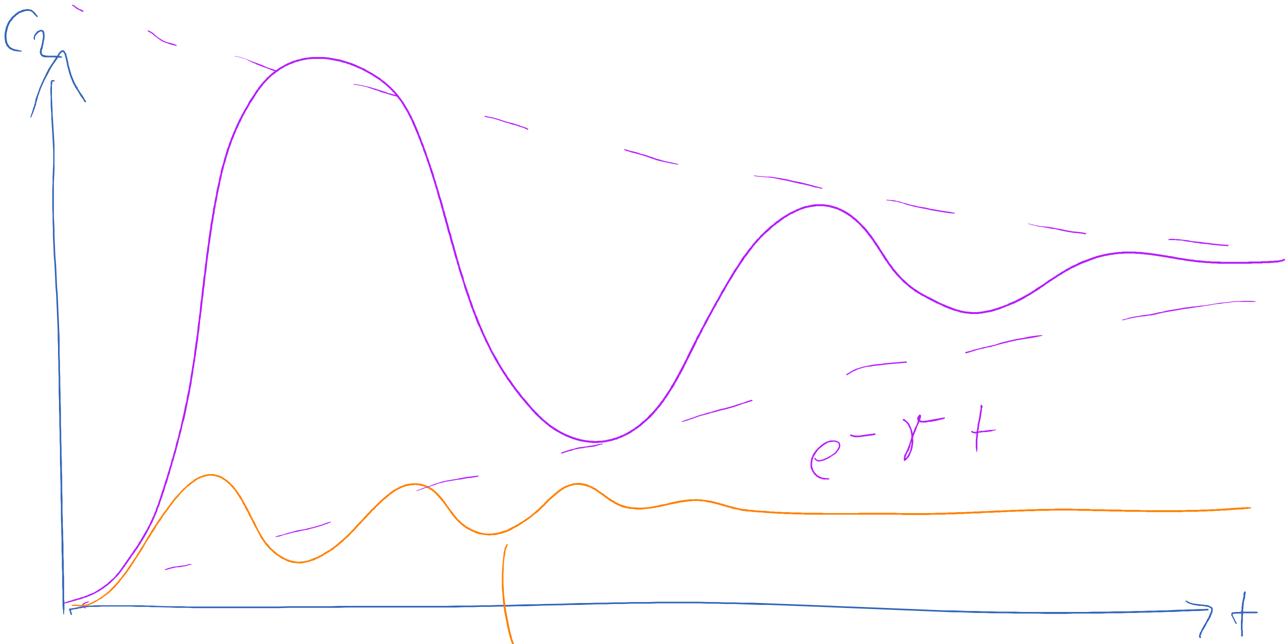
$$W = \sqrt{\Omega^2 + \delta^2}$$

$$\left(\Omega = \sqrt{\Omega_0^2 + \delta^2} \text{ usually} \right)$$

generalized Rabi frequency



with Damping $(\Gamma > 0)$

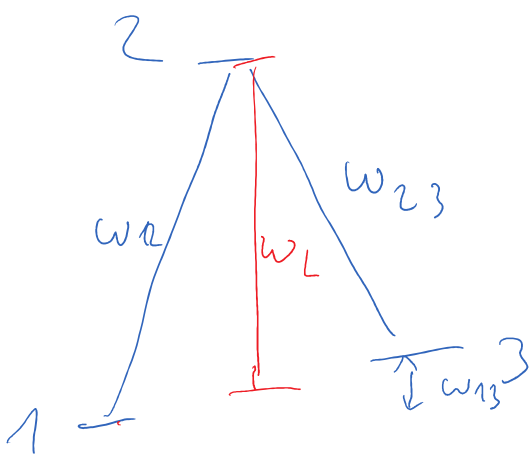


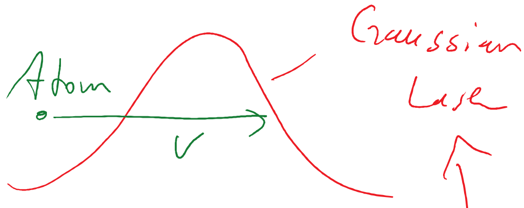
detuning + Damping

e.g. 3 Level system

$$\rho = \begin{pmatrix} & 3 \times 3 & \end{pmatrix}$$

ω_L





$E_0 = E_0(t)$

$\rightarrow \Omega = \Omega(t)$

1 $\underline{}$ $\underline{}$ $\updownarrow \omega_{13}$

$$E(t) = \underline{\underline{E_0(t)}} \cos \omega_L \cdot t$$