OBEs and the Bloch sphere

Donnerstag, 22. Dezember 2022 12:13

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12:46

(7.34, 35)

$$\tilde{c}_1 = c_1 e^{-i\frac{\delta}{2}t}$$
$$\tilde{c}_2 = c_2 e^{+i\frac{\delta}{2}t}$$

$\delta = \omega - \omega_0$
detuning

density matrix

S.E. coefficients

D.M. coefficients

$$\rho = \boxed{\begin{pmatrix} \overset{|c_1|^2}{c_1 c_1^*} & c_1 c_2^* \\ c_1^* c_2 & \underset{|c_2|^2}{c_2 c_2^*} \end{pmatrix}} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$

coherences populations

population: probability to find atom in state $|1\rangle$ or $|2\rangle$

coherences: are related to the phase between the states

in particular, if $\rho_{12} = \rho_{21} = 0$
the phase between $|1\rangle$ and $|2\rangle$ is undefined
$\rightarrow$ statistical mixture of atoms

Example: $c_1 = c_2 = \frac{1}{\sqrt{2}}$

$$\rightarrow \rho = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

populations = 50:50

coherences give a well-defined phase between the states

$$\Psi(\vec{r},t) = \underline{c_1 e^{-i\omega_1 t} u_1(\vec{r})} + \underbrace{c_2 e^{i\varphi}} \underline{e^{-i\omega_2 t} u_2(\vec{r})}$$

time evolution of the elements of the density matrix gives the physics going on

$$\dot{\rho}_{11} = i\frac{\Omega_0}{2}\left(\tilde{\rho}_{21} - \tilde{\rho}_{12}\right)$$

$\Omega_0 = \Omega$

Ω = Rabi frequency

$$\dot{\rho}_{22} =$$

Foot (7.42)

$$\dot{\rho}_{12} =$$

$$\dot{\rho}_{21} =$$

We can introduce incoherent processes (e.g. spontaneous decay $|2\rangle \to |1\rangle$):

$$\dot{\rho}_{11} = \boxed{\gamma\rho_{22}} \boxed{+ i\frac{\Omega_0}{2}\left(\tilde{\rho}_{21} - \tilde{\rho}_{12}\right)}$$

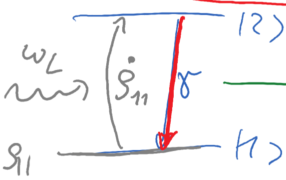
$$\dot{\rho}_{22} = \boxed{-\gamma\rho_{22}} \boxed{+ i\frac{\Omega_0}{2}\left(\tilde{\rho}_{12} - \tilde{\rho}_{21}\right)}$$

$$\dot{\tilde{\rho}}_{12} = -\left(\frac{\gamma}{2} + i\delta\right)\tilde{\rho}_{12} + i\frac{\Omega_0}{2}\left(\rho_{22} - \rho_{11}\right)$$

$$\dot{\tilde{\rho}}_{21} = -\left(\frac{\gamma}{2} - i\delta\right)\tilde{\rho}_{21} + i\frac{\Omega_0}{2}\left(\rho_{11} - \rho_{22}\right)$$

interaction with L_{ij}

Spont. decay



$\gamma \equiv \Gamma_{Foot}$

Factor $\frac{1}{2}$ is motivated in 7.5.1

OBE

The Bloch sphere

$\rho_{11} + \rho_{22} = 1$

u, v, w

7.41, 45, 46 $\to$ (7.67)

$$\dot{u} = \delta \cdot v - \frac{\Gamma}{2}u$$

same

$$\dot{v} = -\delta u + \Omega w - \frac{\Gamma}{2}v$$

$$\dot{w} - \Omega v - \Gamma(w-1)$$

$\Gamma = \gamma$ spontaneous decay rate

(7.67)

$w = \cancel{\rho_{11} - \rho_{22}}$ inversion

$w = \rho_{22} - \rho_{11}$

change

7.54 $\cancel{w = \rho_{11} - \rho_{22}}$

$\cancel{g.s.\ w = +1}$

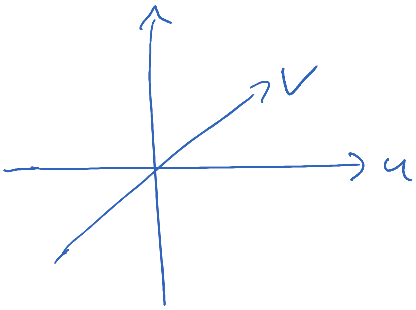
$\cancel{e.s\ w = -1}$

$|2\rangle$

$|1\rangle$

$w = \rho_{22} - \rho_{11}$

Bloch sphere w



change sign of this one

e.g. $\cancel{w = -1}$

o.k., but not standard

But: (7.74)

$$w = \frac{N_2 - N_1}{N} = \rho_{22} - \rho_{11}$$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -\Omega \\ 0 \\ -\delta \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} - \underbrace{\overset{=\gamma}{\Gamma} \begin{pmatrix} u/2 \\ v/2 \\ w+1 \end{pmatrix}}_{\text{Damping}}$$

in standard notation

$w = \rho_{22} - \rho_{11}$

u, v, w are components of the Bloch vector

$$u = \tilde{\rho}_{12} + \tilde{\rho}_{21} = 2\,\mathrm{Re}\,(\tilde{\rho}_{12}) \quad \text{dispersive}$$
$$v = i(\tilde{\rho}_{21} - \tilde{\rho}_{12}) = 2\,\mathrm{Im}\,(\tilde{\rho}_{12}) \quad \text{absorptive}$$

component of Bloch vector

$w = \rho_{22} - \rho_{11}$ inversion

$w = -1$ all in g.s.

$w = +1$ excited state

$$+ \boxed{\rho_{11} + \rho_{22} = 1}$$

$\Rightarrow$ can rewrite the OBE,

$$\dot{u} = \delta v - \frac{\gamma}{2} u$$
$$\dot{v} = -\delta u + \Omega_0 w - \frac{\gamma}{2} v$$
$$\dot{w} = -\Omega_0 v - \gamma (w + 1)$$

Foot:

$\Omega\,(\text{Foot}) = \Omega_0\,(\text{here})$

$\Gamma\,(\text{Foot}) = \gamma\,(\text{here})$

on-resonance Rabi frequency

$$\dot{w} = -\Omega_0 v \quad - \gamma (w+1)$$ Rabi frequency

$\hat{=}$

$$\frac{d}{dt}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = - \begin{pmatrix} \Omega_0 \\ 0 \\ \delta \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} - \gamma \begin{pmatrix} u/2 \\ v/2 \\ w+1 \end{pmatrix}$$

Bloch vector

Damping

Precession of Bloch vector around axis given by $\begin{pmatrix} -\Omega_0 \\ 0 \\ -\delta \end{pmatrix}$ ⇐ 7.2?

with angular frequency $\Omega = \sqrt{\Omega_0^2 + \delta^2}$

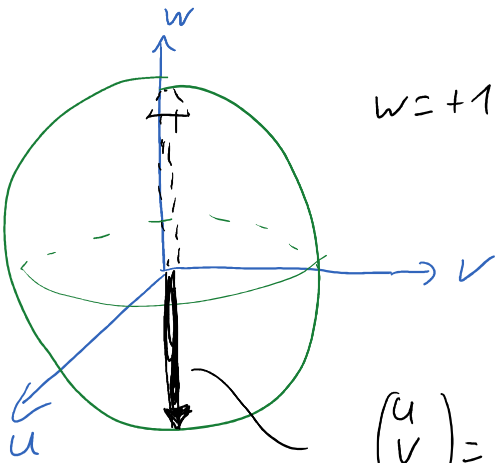
called W in Foot

generalized Rabi frequency

without damping $(\gamma = 0)$: $u^2 + v^2 + w^2 = 1$

→ Bloch vector is moving on a unit sphere

Examples:

① 

$w = +1$ ⇒ all atoms in excited state

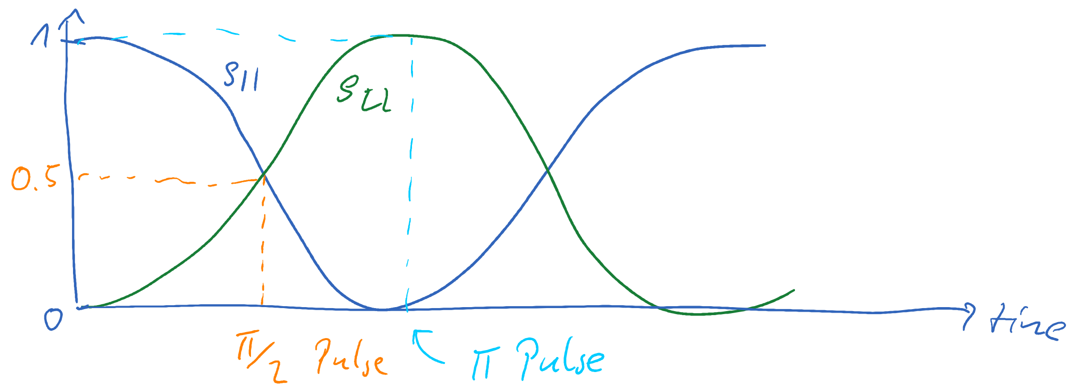
$w = \rho_{22} - \rho_{11}$

$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$ all atoms in the ground state

↯ Foot's definition

② Rabi oscillations:

all atoms in g.s., then on-resonance ($\delta=0$) pulse



$\pi/2$ Pulse: coherent superposition of g.s & e.s.

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$\tilde{c}_1 = \cos\left(\frac{1}{2}\underbrace{\Omega_0 t}_{\pi/2}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\tilde{c}_2 = i\sin\left(\frac{1}{2}\Omega_0 t\right) = i\sin\frac{\pi}{4} = i\frac{1}{\sqrt{2}}$$

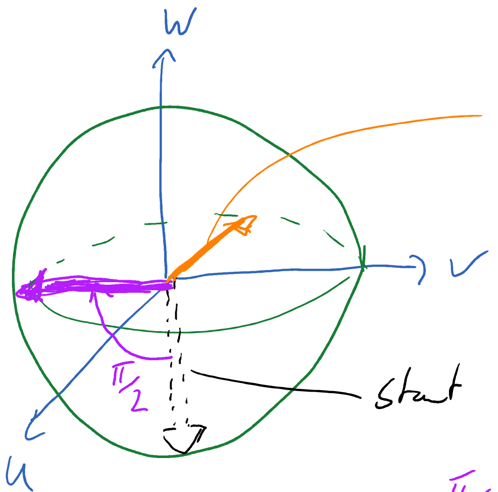
$$\rho_{11} = \tilde{c}_1\tilde{c}_1^* = \frac{1}{2} \qquad \rho_{12} = \tilde{c}_1\tilde{c}_2^* = -\frac{i}{2}$$

$$\rho_{22} = \tilde{c}_2\tilde{c}_2^* = \frac{1}{2} \qquad \rho_{21} = \tilde{c}_2\tilde{c}_1^* = \frac{i}{2}$$

$$u = \rho_{12} + \rho_{21} = -\frac{i}{2} + \frac{i}{2} = \underline{u = 0}$$

$$v = i(\rho_{21} - \rho_{12}) = i\left(\frac{i}{2} + \frac{i}{2}\right) = \underline{v = -1}$$

$$w = \rho_{22} - \rho_{11} = 0$$



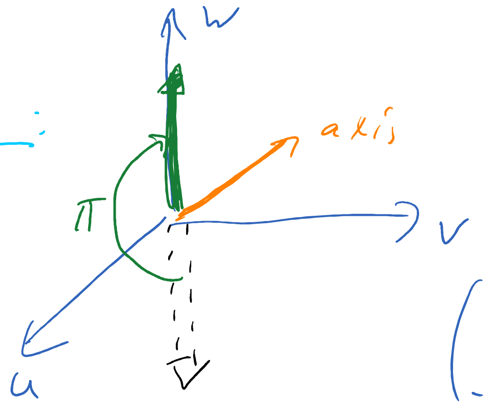
axis $\begin{pmatrix} -\Omega_0 \\ 0 \\ 0 \end{pmatrix}$

start g.s. $\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

$+\pi/2$ around axis $\begin{pmatrix} -\Omega_0 \\ 0 \\ -\delta = 0 \end{pmatrix}$

Here we're looking on resonance

π Pulse:



π Pulse

$$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Rabi oscillations

resonant : $\delta = 0$

Bloch vector is continuosly rotating

around axis $\begin{pmatrix} -\Omega_0 \\ 0 \\ 0 \end{pmatrix}$

b) off-resonant excitation $\delta \neq 0$

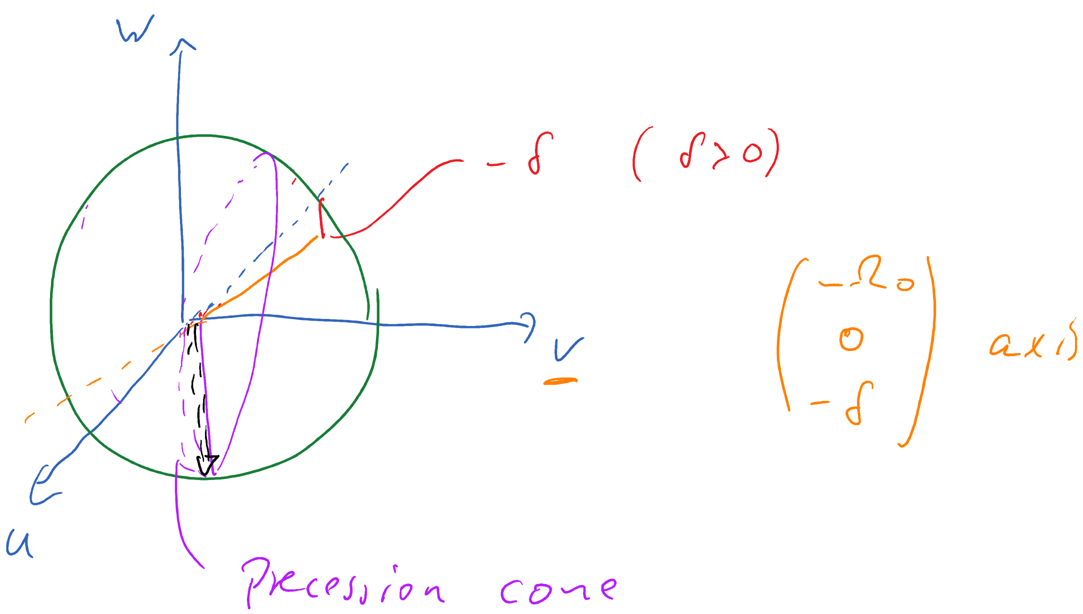
rotation axis $\neq$ u axis

but $(-\Omega_0, 0, -\delta)$ ↳ component along w (z axis)

Rabi frequency

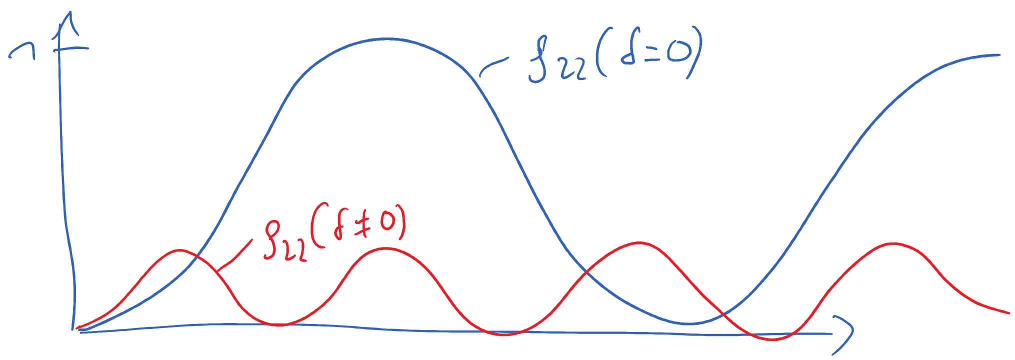
$$\Omega = \sqrt{\Omega_0^2 + \delta^2}$$

W (Foot)



$\delta \neq 0 \implies$ $w = +1$ can not be reached!

oscillation faster $\left(\Omega = \sqrt{\Omega_0^2 + \delta^2}\right)$

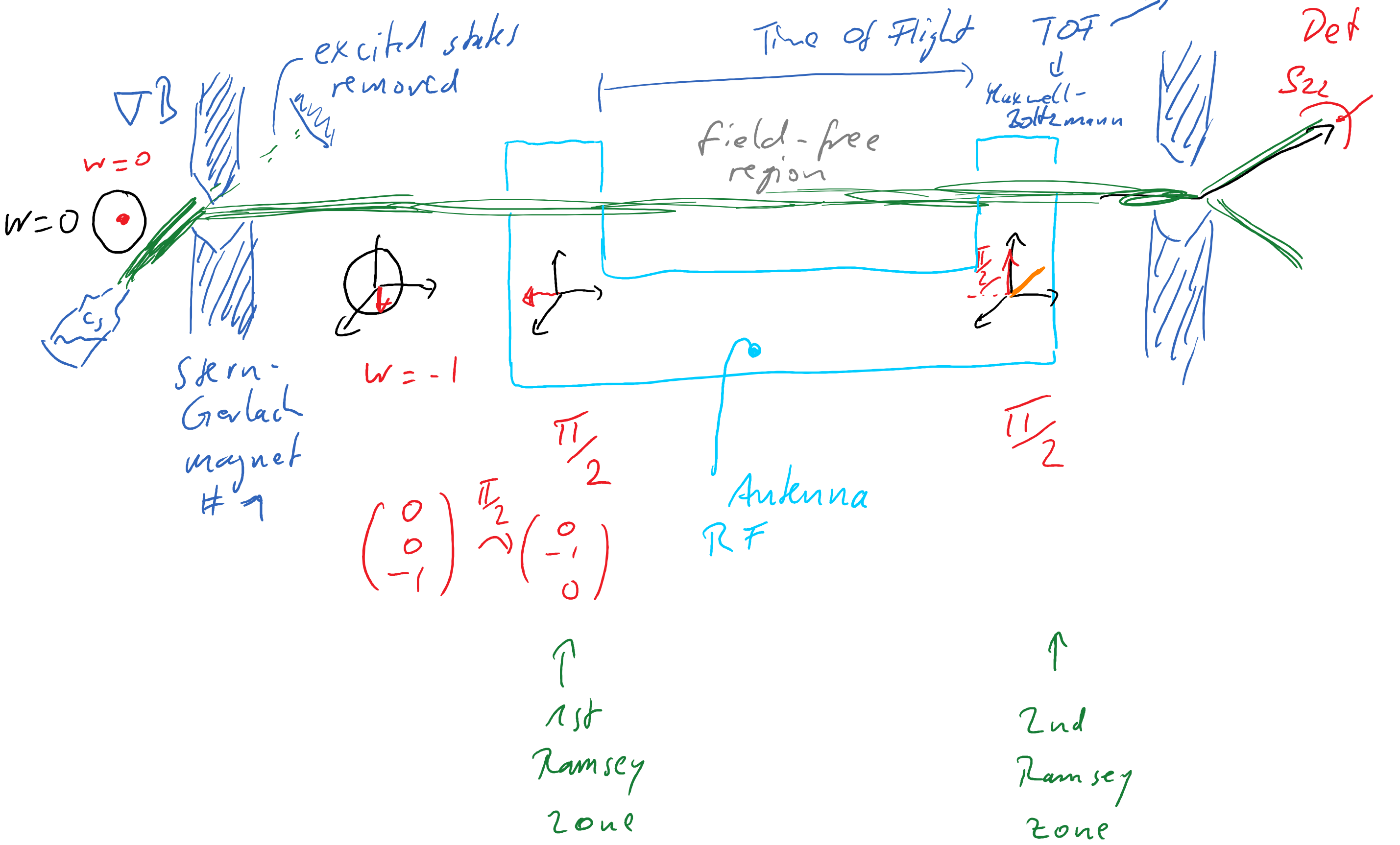


Ramsey apparatus : Ramsey = Separated Oscillatory

Fields

SOF

commonly used in Cs clocks



sinc^2 envelope

quick modulation with $\cos^2\left(\frac{\delta T}{2}\right)$

(7.52)