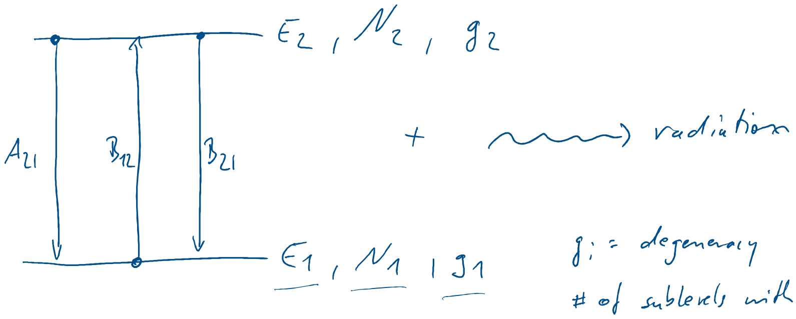
Light - atom - interaction

Einstein A & B coefficients

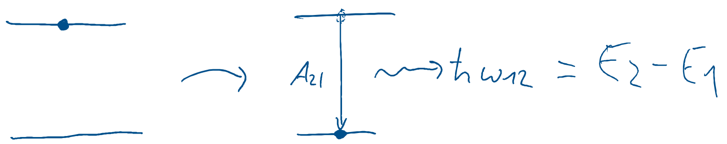
$\boxed{\text{Foot p. 11}}$

2-level atom (toy atom) + e.m. radiation

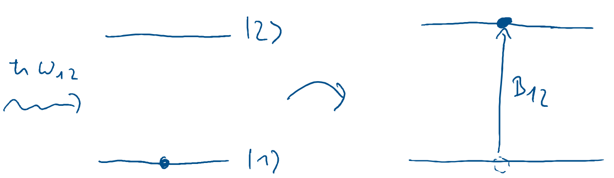


g_i = degeneracy
# of sublevels with the same energy

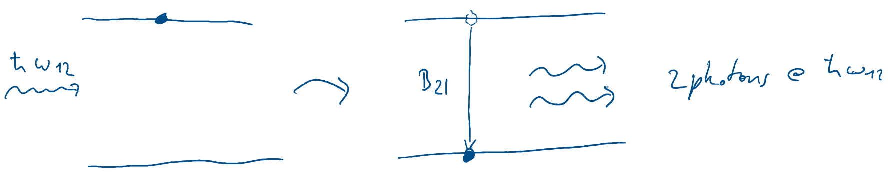
A coefficient: spontaneous decay



B coefficients: Absorption



stimulated emission



$\hbar\omega_{12} = E_2 - E_1 \quad \hat{=}$ resonant light (1)

interaction of atom with radiation field

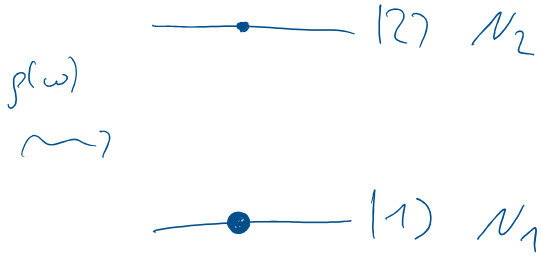
↳ $\rho(\omega)$ = energy density (per unit frequency interval)

$B_{12} \propto \rho(\omega_{12})$

A_{21} is nonzero even if there is no ext. light field

↳ for $\rho(\omega_{12}) = 0$

B_{21} from B_{12} by symmetry



rate equations:

absorption $\qquad$ stim. emission $\qquad$ spont. emissio

- $$\frac{dN_2}{dt} = + N_1 \cdot B_{12} \cdot \rho(\omega_{12}) - N_2 \cdot B_{21} \cdot \rho(\omega_{12}) - N_2 \cdot A_{21} \qquad (2)$$

- $$\frac{dN_1}{dt} = - \frac{dN_2}{dt} \qquad (3)$$ because $N_1 + N_2 = 1 = \text{const.}$

① no laser ; $\rho(\omega_{12}) = 0$

but $N_2(t=0) \neq$ $\quad$ some excited state pop. exists @ $t=0$

$$\frac{dN_2}{dt} = - N_2 A_{21}$$

$$\rightarrow N_2(t) = N_2(0) \cdot e^{-A_{21} \cdot t}$$ exponential decay $\quad (4)$

mean lifetime of upper state $\quad \boxed{\frac{1}{\tau} = A_{21}} \quad (5)$

- Einstein's argument about A & B

put atom in a heated box

$\rightarrow$ black-body radiation

$\rho(\omega)\,d\omega$ depends on temperature

Planck's law

$$\rho(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3} \frac{1}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1} \qquad (6)$$

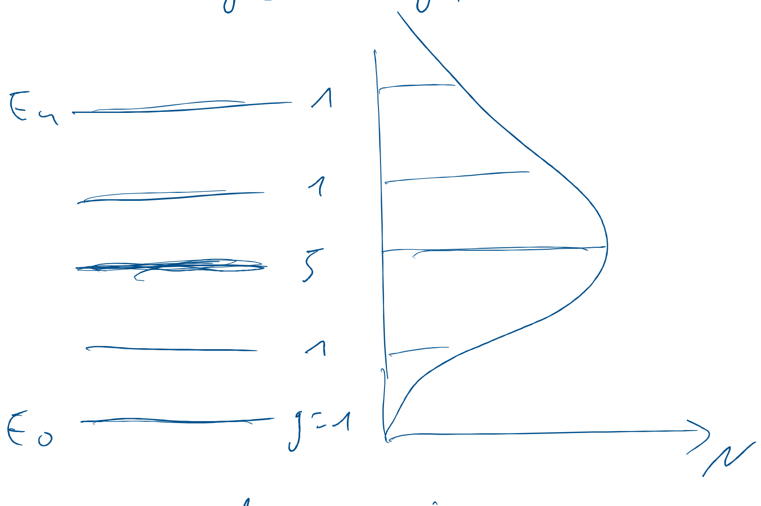
eventually the atom will get into equilibrium with the field

$$\rightarrow \frac{dN_i}{dt} = 0$$

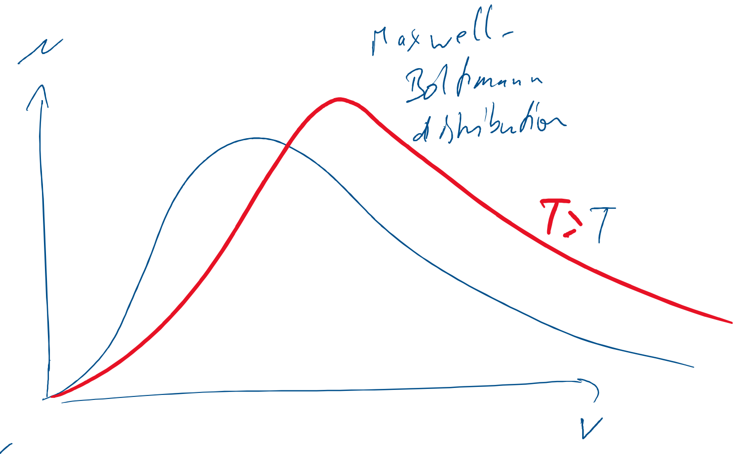
from (2): $$\rho(\omega_{12}) = \frac{A_{21}}{B_{21}} \frac{1}{\frac{N_1}{N_2} \cdot \frac{B_{12}}{B_{21}} - 1} \qquad (7)$$

- populations in thermal equilibrium are given by the Boltzmann factor

$$\frac{N_2}{g_2} = \frac{N_1}{g_1} \exp\left(-\frac{\hbar\omega_{12}}{kT}\right) \qquad (8)$$



degeneracies

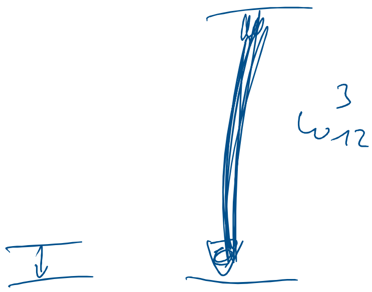


(6) ... (8) hold for every temperature

$\to$ 2 equations, 1 for T independent part 0

1 for part $e^{-\frac{\hbar\omega_{12}}{kT}}$

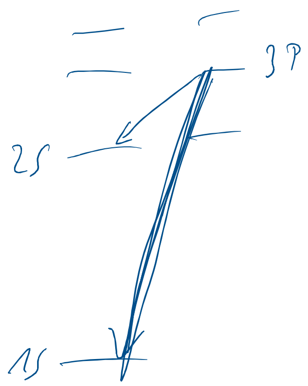
$$\boxed{\begin{aligned} A_{21} &= \frac{\hbar\,\omega_{12}^3}{\pi^2 c^3}\, B_{21} \\ B_{12} &= \frac{g_2}{g_1}\cdot B_{21} \end{aligned}} \qquad (9)$$



Note $A_{21} \propto \omega_{12}^3$!

spontaneous decay rate
is proportional to the 3rd power of the energy

e.g. $\boxed{H}$



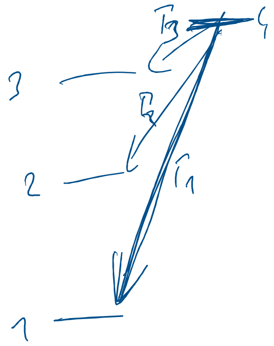
- Einstein coefficients are properties of the <u>atom</u>
- they hold for any kind of radiation
  Black body, monochromatic,
  monochromatic
- strong absorption is associated with strong emission

Note about (5) $\boxed{\frac{1}{\tau} = A_{21}}$

is true

for real atoms τ = total lifetime of a state

$\frac{1}{\tau} = \Gamma_{tot}$ total decay rate



$\Gamma_{tot} = \Gamma_1 + \Gamma_2 + \Gamma_3$

$\uparrow$

"A_{21}"

A_{41}