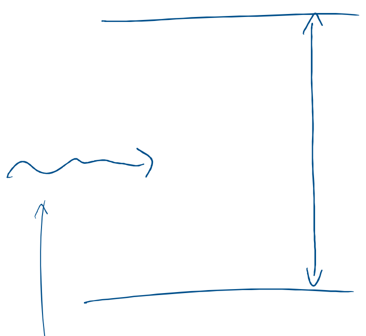
Light-atom interaction

Metcalf + van der Straten



$|2\rangle \quad E_2 = \hbar\omega_2$

energy difference $\hbar\omega_{12} = \hbar(\omega_2 - \omega_1) = \frac{E_2 - E_1}{\hbar}$

$|1\rangle \quad E_1 = \hbar\omega_1$

light field

(angular) frequency ω_L

- total Hamiltonian $H = H_{Atom} + V(t)$

  H_{Atom}: free atom, no light

  $V(t)$: interaction

- interaction $V = -\vec{d} \cdot \vec{E}(\vec{r}, t)$

  $\vec{d}$: dipole operator, atomic dipole moment

  $\vec{E}(\vec{r}, t)$: ext. electric field, light field

— semiclassical approximation: light field is <u>not</u> quantized

— "Dipole approximation"

atom is much smaller than wavelength of light field

$\uparrow$

$a_0 = 0.05\,nm$

$\uparrow$ field

100s of nm = vis. light

dimension over which the el. field changes

- atom does not change its position during interaction

$$\Rightarrow \vec{E}(\vec{r},t) = E(\vec{R},t) = E(t) = \vec{\varepsilon}\, E_0 \cos(\omega_L \cdot t)$$

$\vec{R}$: atom position

$\vec{\varepsilon}$: polarization vector

$$\Rightarrow V(t) = -\vec{d}\cdot\vec{E}(t) \quad \text{with} \quad \vec{d} = -e\vec{r}$$

time-dependent SE

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{r},t) = H\,\Psi(\vec{r},t) = \left[H_0 + V(t)\right]\cdot \Psi(\vec{r},t)$$

ansatz

$$\Psi(\vec{r},t) = c_1(t)\cdot e^{-i\omega_1 t}\cdot u_1(\vec{r}) + c_2(t)\, e^{-i\omega_2 t}\, u_2(\vec{r})$$

$u_1(\vec{r})$: spatial w.f.

$e^{-i\omega_1 t}$, $e^{-i\omega_2 t}$: time evolution without field

$c_1(t)$, $c_2(t)$: time-dependent coefficient

into SE:

$$\Rightarrow \partial c_1(t) = \ldots\; i\,\frac{d\,E_0}{}\, e^{-i\omega_{21} t} \cos(\ldots t)\, c_2(t)$$

$$\Rightarrow \frac{\partial c_1(t)}{\partial t} = \dot{c}_1 = i \frac{d E_0}{\hbar} e^{-i\omega_{21} t} \cos(\omega_L t)\, c_2(t)$$

$$\dot{c}_2 = i \boxed{\frac{d E_0}{\hbar}} e^{i\omega_{21} t} \cos(\omega_L t)\, c_1(t)$$

this d = projection of el. dipole matrix element onto light polarization vector

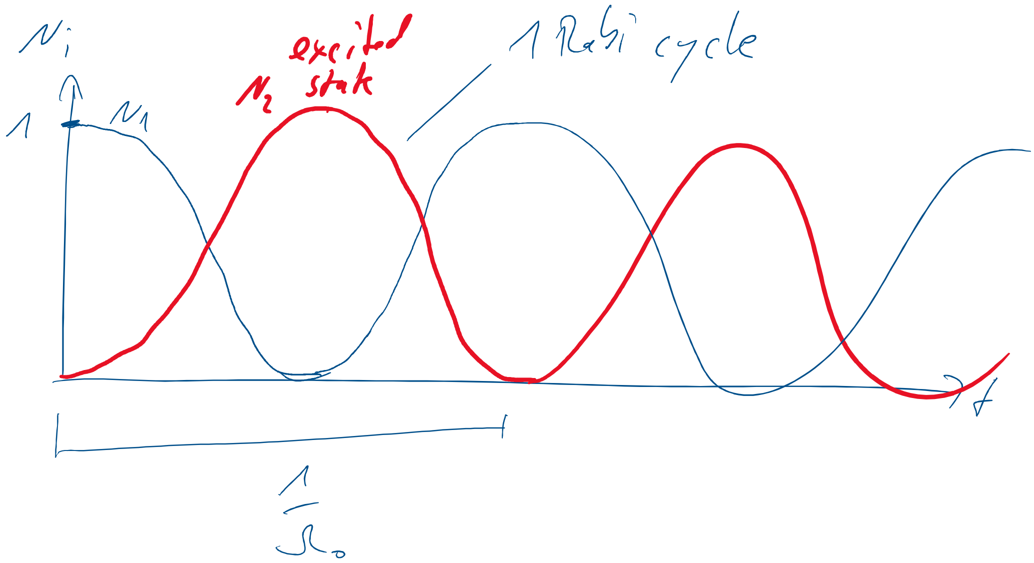
$$d = d_{12}^{\varepsilon} = \langle 1 | \hat{\vec{d}} | 2 \rangle \cdot \vec{\varepsilon}$$

d gives transition strength , $d = 0 \mathrel{\hat{=}}$ transition forbidden

<u>important quantity:</u>

resonant Rabi frequency $\Omega_0 = \frac{d E_0}{\hbar}$

<u>later:</u> Rabi oscillations



$$\rightarrow \dot{c}_1(t) = i \Omega_0 e^{-i\omega_{21} t} \cos \omega_L t \;\; c_2(t)$$

$$\rightarrow \dot{c}_1(t) = i\Omega_0 e^{-i\omega_{21}t} \cos\omega_L t \; c_2(t)$$

$$\dot{c}_2(t) = i\Omega_0 e^{+i\omega_{21}t} \cos\omega_L t \; c_1(t)$$

Euler: $\cos(\omega_L t) = \frac{1}{2}\left(e^{i\omega_L t} + e^{-i\omega_L t}\right)$

$$\rightarrow \dot{c}_1(t) = i\frac{\Omega_0}{2} e^{-i\omega_{21}t}\left(e^{i\omega_L t} + e^{-i\omega_L t}\right) c_2(t)$$

$$\dot{c}_2(t) = i\frac{\Omega_0}{2} e^{i\omega_{21}t}\left(e^{i\omega_L t} + e^{-i\omega_L t}\right) c_1(t)$$

We consider only light close to resonance

$$\omega_L \approx \omega_{21}$$

"Rotating wave approximation" RWA

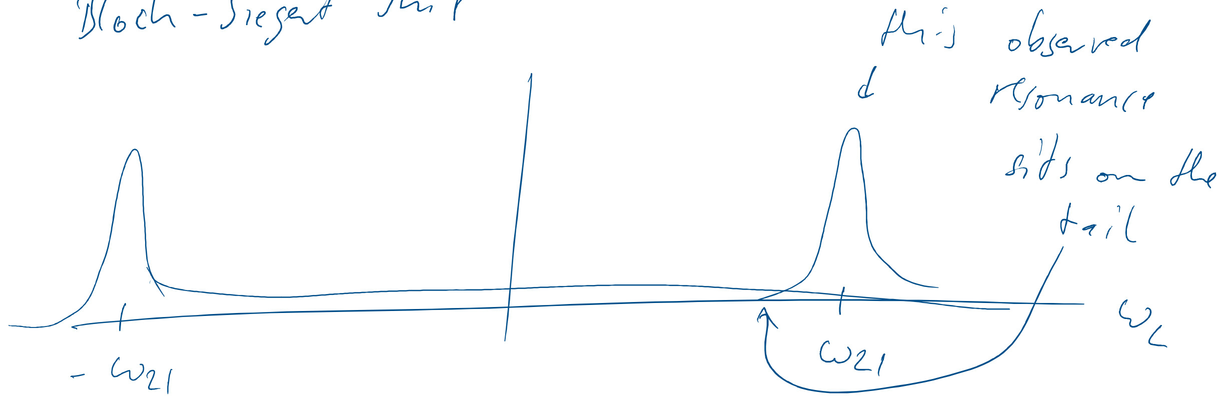
$\rightarrow$ ignore terms oscillating with $\omega_L + \omega_{21}$

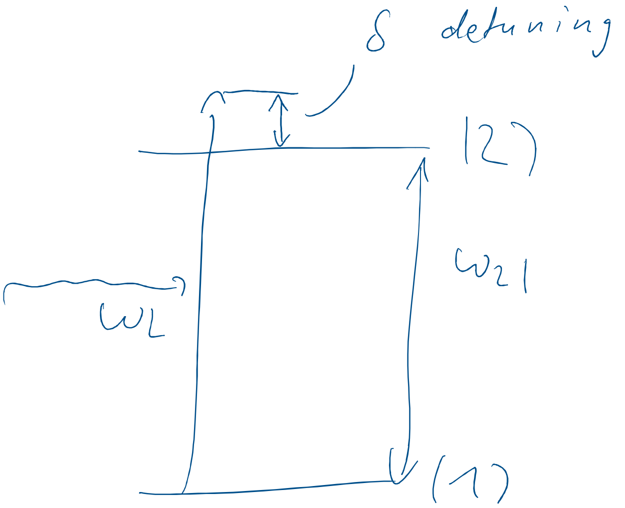
= very fast, averages out quickly

keep $\boxed{\omega_L - \omega_{21} = \delta = \text{Detuning}}$

Note: for very high resolution RWA makes tiny line shifts

Bloch-Siegert shift





$\rightarrow$ optical Bloch equations
density matrix