Density matrix

SE is always coherent

how to include e.g. spontaneous decay?
(or other incoherent processes)

so far: pure states

$$\psi(\vec{r},t) = c_1 e^{-i\omega_1 t} u_1(\vec{r}) + c_2 e^{-i\omega_2 t} u_2(\vec{r})$$

coherent superposition of $|1\rangle$ & $|2\rangle$
c_i are determined by SE

now: <u>statistical mixture</u>

e.g. 50% in $|1\rangle$
50% in $|2\rangle$

$\rightarrow$ incoherent superposition

$\Rightarrow$ density matrix operator

- system with several states
- propability (in statistical sense) to find atom in state $|\psi_k\rangle$ is p_k

$$\hookrightarrow \hat{\rho} = \sum_i p_k |\psi_k\rangle\langle\psi_k|$$

elements of the density matrix

$$\rho_{ij} = \langle i|\hat{\rho}|j\rangle$$

DM can describe pure states and statistical mixtures

2 level system

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \begin{pmatrix} c_1 c_1^* & c_1 c_2^* \\ c_2 c_1^* & c_2 c_2^* \end{pmatrix}$$

populations

coherences

↳ related to the phase relation between states

if coherence $\rho_{ik} = 0$

↳ phase between $|i\rangle$ & $|k\rangle$ is undefined

e.g. $c_1 = c_2 = \frac{1}{\sqrt{2}} \rightarrow \rho = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

coherences $\neq 0$

well-defined phase between $|1\rangle$ & $|2\rangle$

↳ constant in time ⟶ φ = constant

↳ constant in time $\longrightarrow$ φ = constant

$$\psi(\vec{r},t) = |c_1|\, e^{-i\omega_1 t}\, u_1(\vec{r}) + |c_2|\, e^{i\varphi}\, e^{-i\omega_2 t}\, u_2(\vec{r})$$