Optical Bloch Equations (OBE)

time evolution of the elements of DM

$$i\hbar \frac{d}{dt}\hat{\rho} = [\hat{H}, \hat{\rho}]$$

we start with the same situation as before
no decay, no damping

$$\rightarrow \frac{d}{dt}\rho_{11} = \frac{d}{dt}(c_1 c_1^*) = \dot{c}_1 c_1^* + c_1 \dot{c}_1^*$$

$$= i\frac{\Omega_0}{2}\left(e^{i\delta t}\rho_{21} - e^{-i\delta t}\rho_{12}\right)$$

↑ c_1 from above

⋮

RWA

$\rho_{11} = \tilde{\rho}_{11}$ $\tilde{\rho}_{12} = e^{-i\delta t}\rho_{12}$

$\rho_{22} = \tilde{\rho}_{22}$ $\tilde{\rho}_{21} = e^{i\delta t}\rho_{21}$

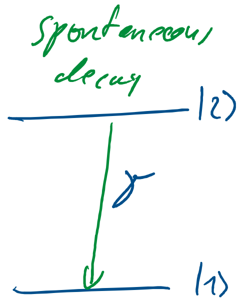
Differential equations for DM elements

$$\dot{\rho}_{11} = \gamma \rho_{22} + i \frac{\Omega_0}{2} \left(\tilde{\rho}_{21} - \tilde{\rho}_{12} \right)$$

$$\dot{\rho}_{22} = -\gamma \rho_{22} + i \frac{\Omega_0}{2} \left(\tilde{\rho}_{12} - \tilde{\rho}_{21} \right)$$

$$\dot{\tilde{\rho}}_{12} = -\left(\frac{\gamma}{2} + i\delta \right) \tilde{\rho}_{12} + i \frac{\Omega_0}{2} \left(\rho_{22} - \rho_{11} \right)$$

$$\dot{\tilde{\rho}}_{21} = -\left(\frac{\gamma}{2} - i\delta \right) \tilde{\rho}_{21} + i \frac{\Omega_0}{2} \left(\rho_{11} - \rho_{22} \right)$$



OBEs for 2 level system

spont. decay introduced phenomenologically

- population is transferred from $|2\rangle \rightarrow |1\rangle$ with rate γ
- coherences are damped with rate $\gamma/2$

$\gamma = \frac{1}{\tau}$ lifetime of $|2\rangle$

we see

$\dot{\rho}_{11} = -\dot{\rho}_{22} \Rightarrow \rho_{11} + \rho_{22} = 1 = \text{const}$

can use this to reduce # of OBEs from $4 \rightarrow 3$

$\rho_{12} = \rho_{21}^*$

we introduce u, v, w

$$\boxed{w = \rho_{22} - \rho_{11}}$$ inversion

$w = 1 \rightarrow$ all atoms in $|2\rangle$

$= 0$ 50:50

$= -1$ all in $|1\rangle$

$$\frac{d}{dt}\tilde{\rho}_{12} = \left(-\frac{\gamma}{2} - i\delta\right)\tilde{\rho}_{21} - \frac{i w \Omega_0}{2}$$

$$\frac{d}{dt} w = -\gamma (w+1) - i\,\Omega_0 \left(\tilde{\rho}_{21} - \tilde{\rho}_{12}\right)$$

dynamics

(1) $t \ll \frac{1}{\gamma}$ damping plays no role

Rabi oscillations $\Omega_0 \gg \frac{1}{\gamma}$

(2) $t \gg \frac{1}{\gamma} \rightarrow$ steady-state solutions

coherences die out