The Optical Bloch Equations (OBEs)

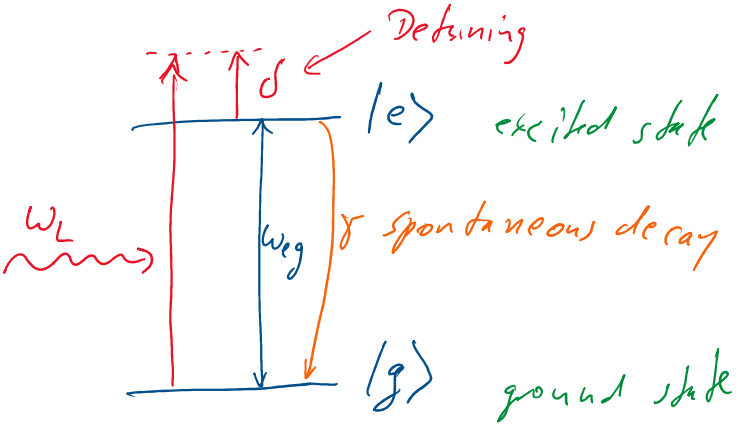
Donnerstag, 4. Mai 2023 13:51

now with correct signs following
van der Straten & Metcalf:
"Atoms & Molecules Interacting with Light"

The DM depends on the Hamiltonian

as $i\hbar \frac{d\rho}{dt} = [\hat{H}, \rho]$ (6.9)

2-level system



$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix} = \begin{pmatrix} c_e c_e^* & c_e c_g^* \\ c_g c_e^* & c_g c_g^* \end{pmatrix} \quad (6.10)$$

↑ ≙ our time-dependent coefficients in the SE treatment

ρ_{ik} from: $\frac{dc_g(t)}{dt} = -i\frac{\Omega^*}{2}\tilde{c}_e(t)$ (2.6a)

$\frac{dc_e(t)}{dt} = -i\frac{\Omega}{2}c_g(t) + i\delta\tilde{c}_e(t)$ (2.6b)

where $\tilde{c}_e(t) \equiv c_e(t)\, e^{i\delta t}$ (RWA)

Note: In this Book's treatment, the Rabi frequency Ω

Note: In this Book's treatment, the Rabi frequency Ω can be a complex number (thus the Ω^* above)

For a cw laser, Ω is usually real.

However, if 2 or more lasers interact, a phase shift can occur, which can be implemented here making Ω complex.

(see p. 18)

We get, e.g. for ρ_{gg}:

$$\frac{d\rho_{gg}}{dt} = \frac{dc_g}{dt} c_g^* + c_g \frac{dc_g^*}{dt} = -i \frac{\Omega^*}{2} \tilde{\rho}_{eg} + i \frac{\Omega}{2} \tilde{\rho}_{ge} \qquad (6.11)$$

Now we introduce the spontaneous decay

$$\left(\frac{d\rho_{eg}}{dt}\right)_{spont.} = -\frac{\gamma}{2} \rho_{eg}$$

$$\Rightarrow \quad \begin{aligned}
\dot{\rho}_{gg} &= +\gamma \rho_{ee} + \frac{i}{2}\left(\Omega \tilde{\rho}_{ge} - \Omega^* \tilde{\rho}_{eg}\right) \\
\dot{\rho}_{ee} &= -\gamma \rho_{ee} + \frac{i}{2}\left(\Omega \tilde{\rho}_{eg} - \Omega^* \tilde{\rho}_{ge}\right) \\
\dot{\rho}_{ge} &= -\left(\frac{\gamma}{2} + i\delta\right) \tilde{\rho}_{ge} + \frac{i}{2} \Omega^* \left(\rho_{gg} - \rho_{eg}\right) \\
\dot{\rho}_{eg} &= -\left(\frac{\gamma}{2} - i\delta\right) \tilde{\rho}_{eg} + \frac{i}{2} \Omega \left(\rho_{ee} - \rho_{gg}\right)
\end{aligned} \qquad (6.13)$$

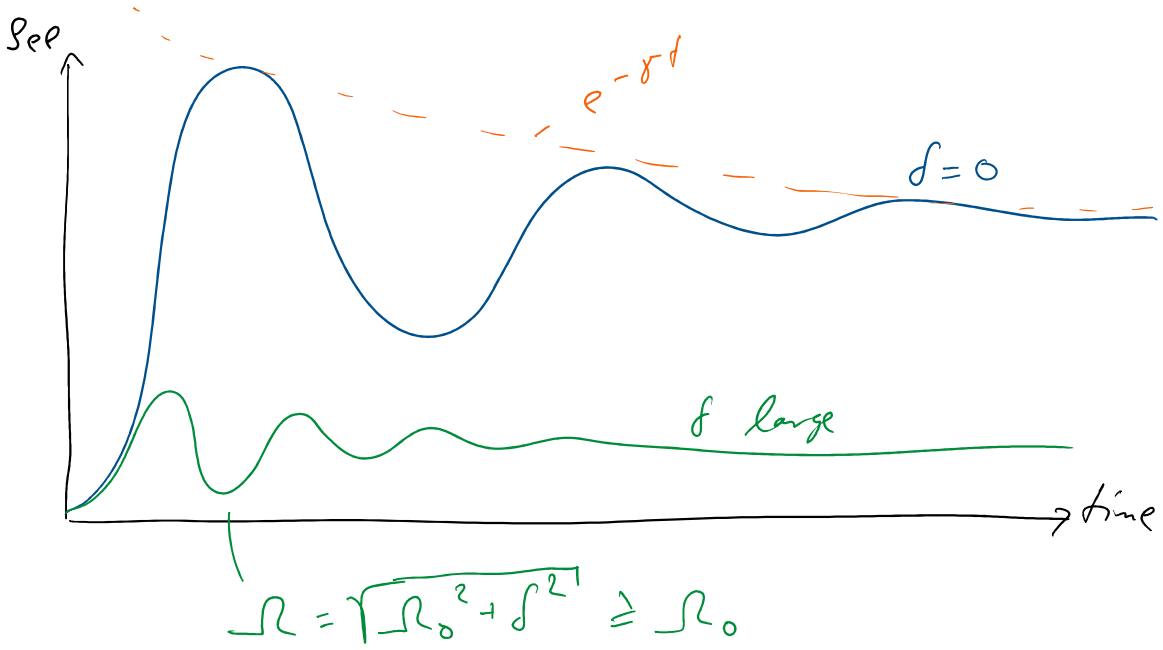
OBEs with correct signs.

$r - 1$ all atoms in $|g\rangle$

... with correct signs.

Now inversion $w = \rho_{ee} - \rho_{gg} = \begin{cases} -1 & \text{all atoms in } |g\rangle \\ 0 & 50:50 \ |e\rangle \ \& \ |g\rangle \\ +1 & \text{all in } |e\rangle \end{cases}$

This is the usual convention for OBEs & the Bloch sphere!



$$\Omega = \sqrt{\Omega_0^2 + \delta^2} \geq \Omega_0$$