Line shape: Power broadening and Saturation

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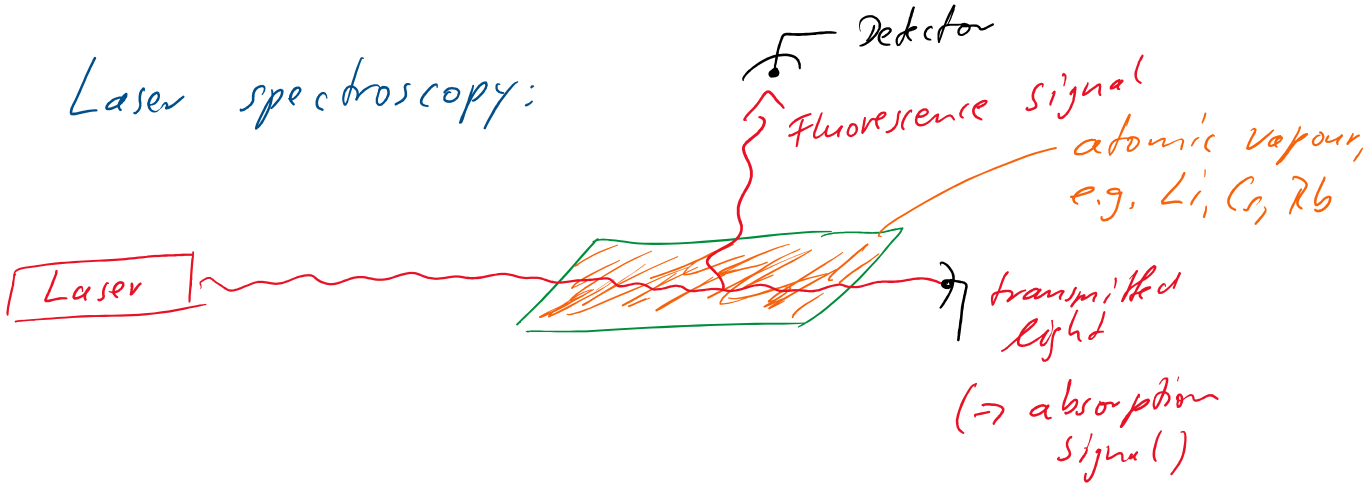
Laser spectroscopy:



The steady-state solutions (at late times, when the oscillations are over) give us the line shape observed in cw laser spectroscopy

↳ continuous wave, i.e. not pulsed laser

The 4 equations of the OBEs are reduced to 3 by $\rho_{gg} + \rho_{ee} = 1$ (atom number conserved) and further to 2 equations, because $\rho_{eg} = \rho_{ge}^*$ are complex conjugates.

We use $w \equiv \rho_{ee} - \rho_{gg}$

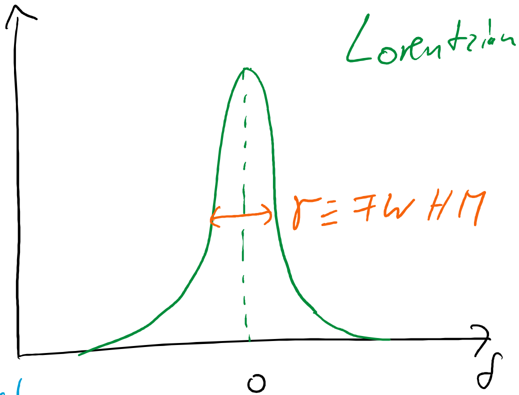
$$\Rightarrow \quad \dot{\rho}_{eg} = -\left(\frac{\gamma}{2} - i\delta\right)\rho_{eg} + \frac{i w \Omega}{2} \qquad (6.14)$$

$$\dot{w} = -\gamma w + i\left(\Omega^* \rho_{eg} - \Omega \rho_{eg}^*\right) - \gamma$$

steady-state $\hat{=}$ $\dot{\rho}_{eg} = 0$ and $\dot{w} = 0$

$$\Rightarrow w = \frac{-1}{1+s} \quad \text{and}$$

$$\rho_{eg} = \frac{-i\Omega}{2\left(\frac{\gamma}{2} - i\delta\right)(1+s)}$$



typical Lorentzian: $y = \frac{1}{x^2 + \frac{FWHM^2}{4}}$

with saturation parameter s

$$s \equiv \frac{|\Omega|^2}{2\left|\left(\frac{\gamma}{2} - i\delta\right)\right|^2} = \frac{|\Omega|^2/2}{\delta^2 + \gamma^2/4} \equiv \frac{s_0}{1 + \left(2\delta/\gamma\right)^2}$$

s_0 = on-resonance saturation parameter

$$s_0 \equiv \frac{2|\Omega|^2}{\gamma^2} = \frac{I}{I_s} \leftarrow \text{Laser intensity}$$

saturation intensity $I_s = \frac{\pi h c}{3 \lambda^3 \tau}$

If s = small $\rightarrow$ $w \approx -1$ (all atoms in g.s.)

s = large $\rightarrow$ $w = 0$ (half-half populations)

$\hookrightarrow \rho_{ee} = 0.5$

$$\rho_{ee} = \frac{1}{2}(1+w) = \frac{s}{2(s+1)} = \frac{s_0/2}{1 + s_0 + \left(\frac{2\delta}{\gamma}\right)^2}$$

The photon scattering rate γ_{ph} is

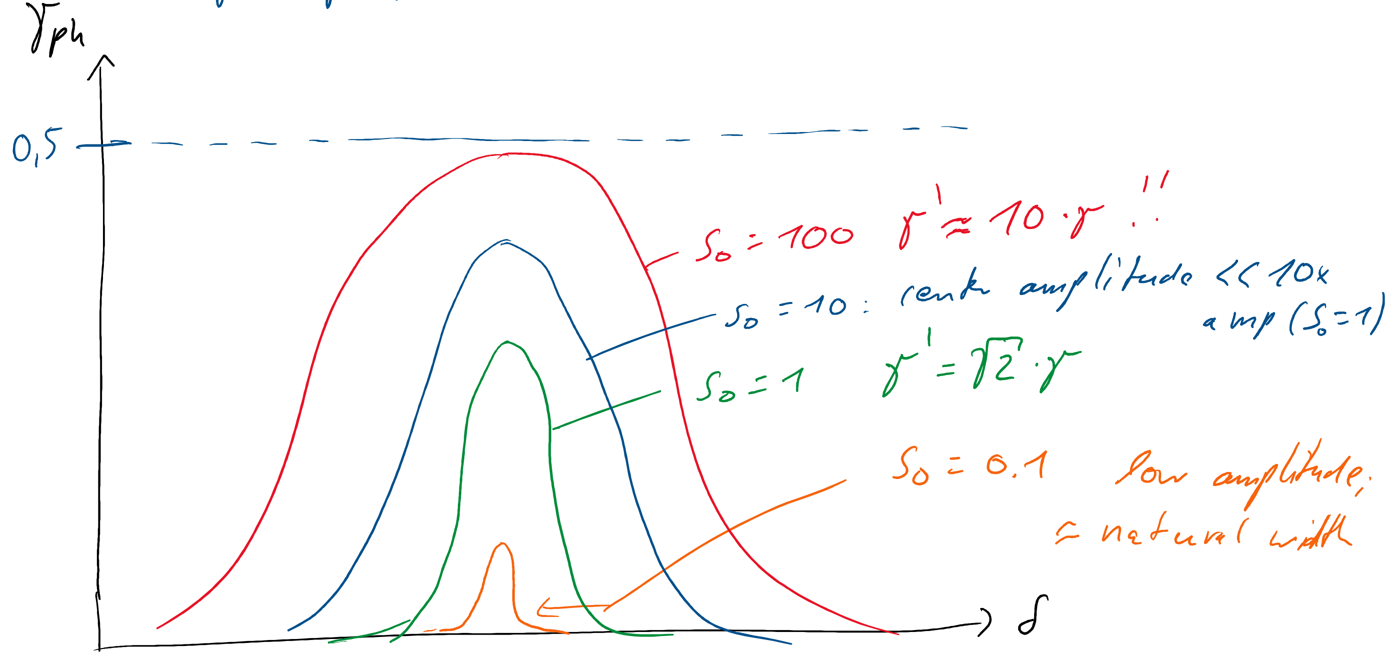
$$\gamma_{ph} = \gamma \cdot \rho_{ee} = \frac{s_0 \gamma/2}{1 + s_0 + \left(\frac{2\delta}{\gamma}\right)^2} \qquad (6.18)$$

Again, for large s_0, $\gamma_{ph} \to \gamma/2$

Rewrite 6.18 $\quad \gamma_{ph} = \left(\frac{s_0}{1+s_0}\right)\left(\frac{\gamma/2}{1 + \left(\frac{2\delta}{\gamma'}\right)^2}\right) \qquad (6.19a)$

with γ' = power-broadened linewidth:

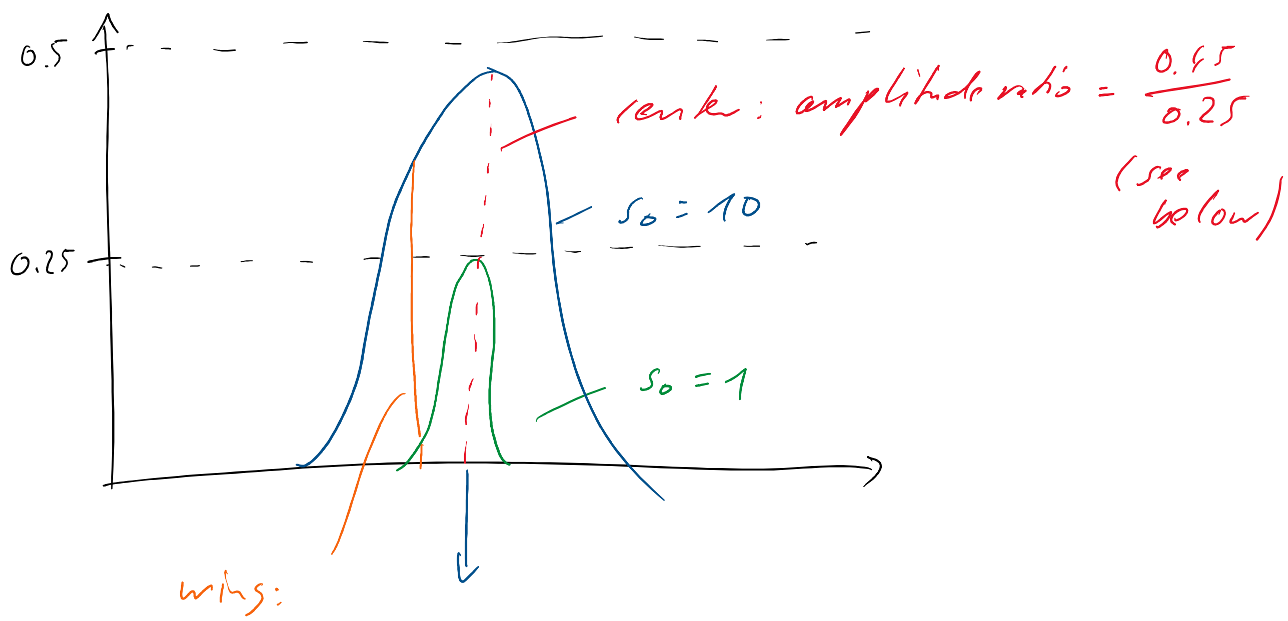
$$\gamma' = \gamma \sqrt{1 + s_0}$$



Saturation: The scattering rate γ_{ph} can never exceed $\frac{\Gamma}{2}$.

In the center, this limit is reached very quickly

In the wings, this happens only for large s_0



see (6.18), $\delta = 0$

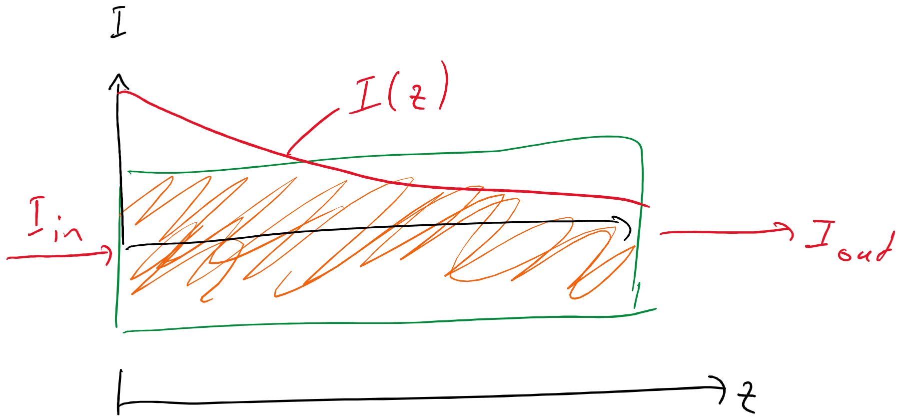
$$\gamma_{ph} = \frac{s_0}{1+s_0} \cdot \frac{\Gamma}{2} \qquad \text{for } \delta = 0$$

$$s_0 = 1 \rightarrow \gamma_{ph}(\delta=0) = \frac{1}{2} \cdot \frac{\Gamma}{2} = 0.25 \cdot \Gamma$$

$$s_0 = 10 \rightarrow \gamma_{ph}(\delta=0) = \frac{10}{11} \cdot \frac{\Gamma}{2} = 0.45\,\Gamma$$

But for $|\delta|$ large ($\hat{=}$ the wings), the signal is still approx. linear in s!

Absorbed light (Beer Lambert law)



$$\frac{dI}{dz} = -\sigma_{eg}\, n\, I$$

number of atoms

scattering cross section (apparent size of an atom)

reduction of intensity because of scattering

$\Rightarrow$ exponential intensity decrease

Result: $$\sigma_{eg} = \frac{\hbar \omega \gamma}{2 I_s} = \frac{3\lambda^2}{2\pi}$$

Bohr radius

The apparent atom size is NOT $\sim a_0^2$ ($a_0 = 50\,\text{pm}$)

but rather λ^2 ($\lambda \approx 500\,\text{nm}$)

$\Rightarrow$ The atom looks HUGE in laser spectroscopy!