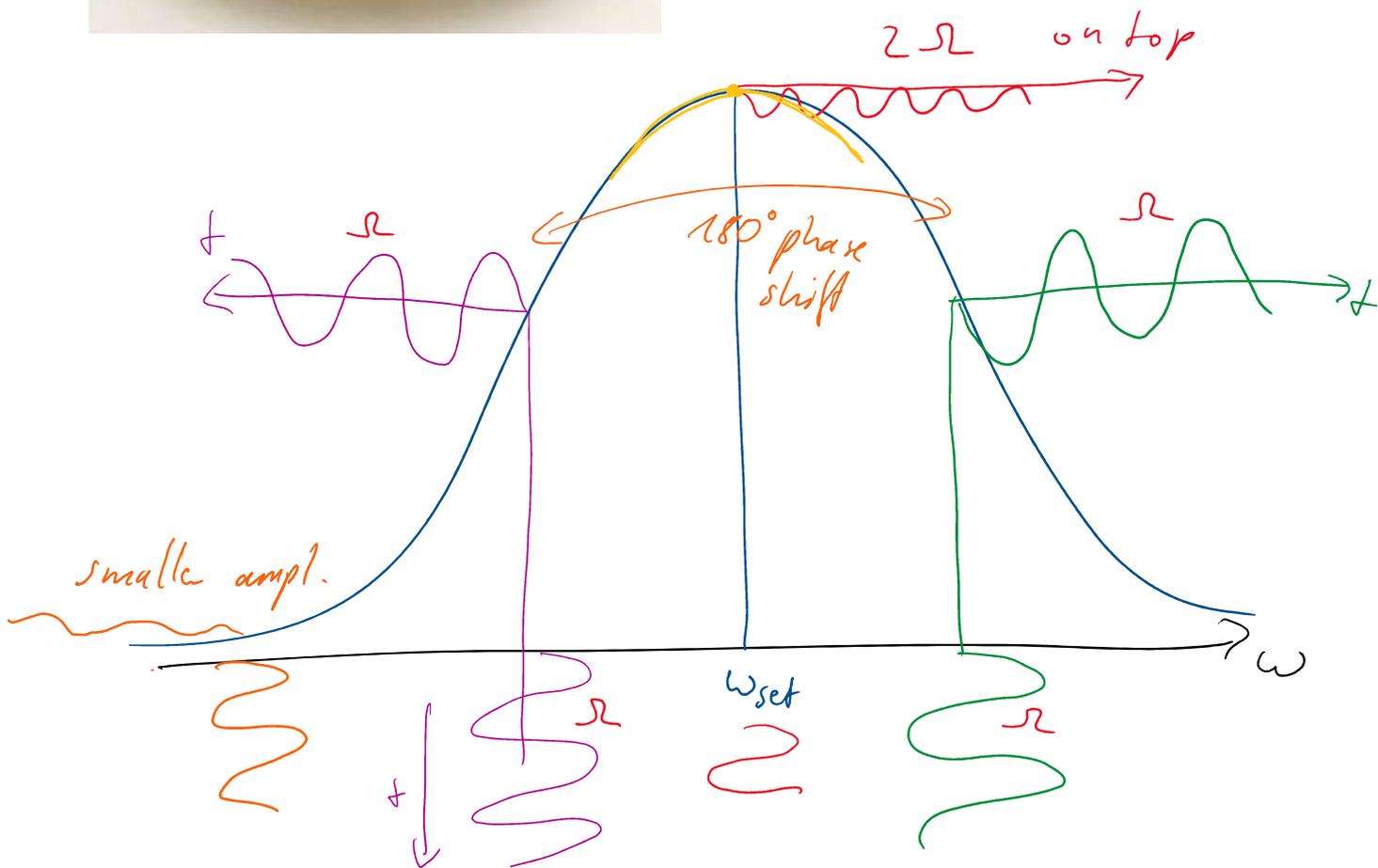
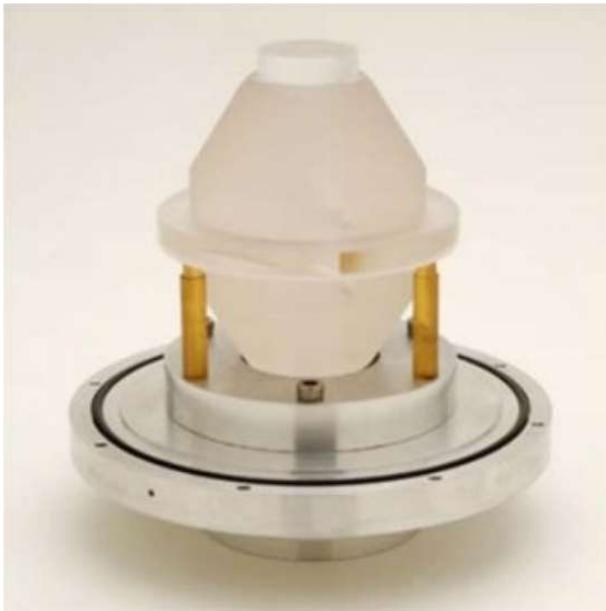


# Pound-Drever-Hall (PDH) Lock

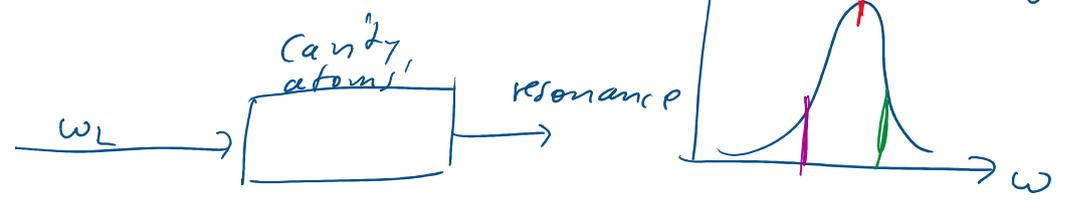
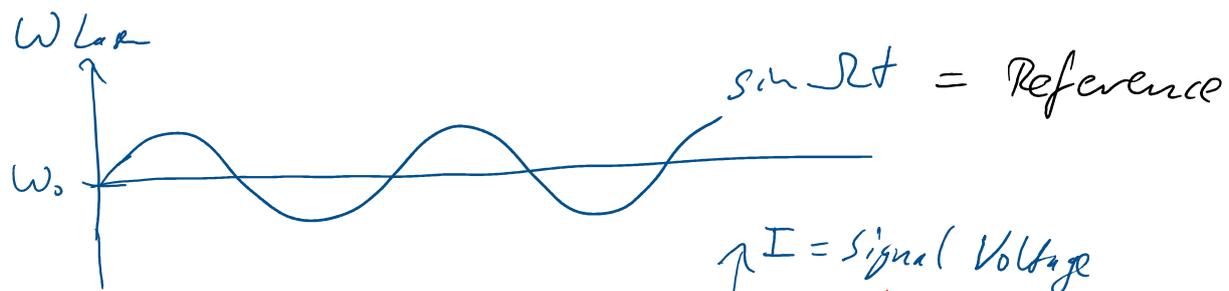
Donnerstag, 1. Juni 2023 12:36



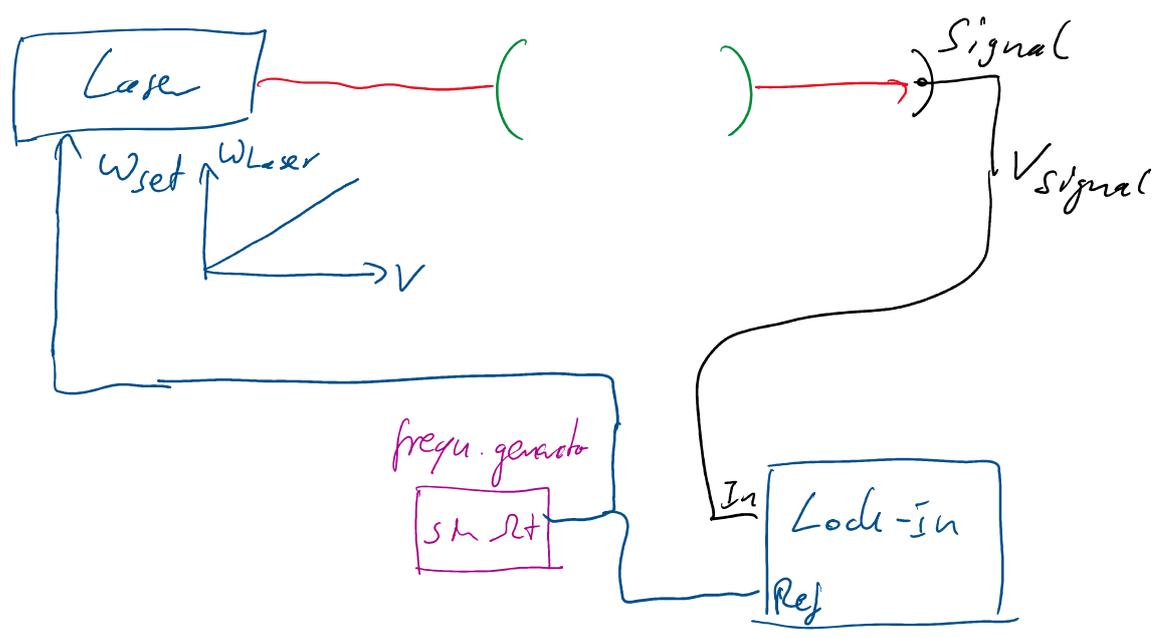
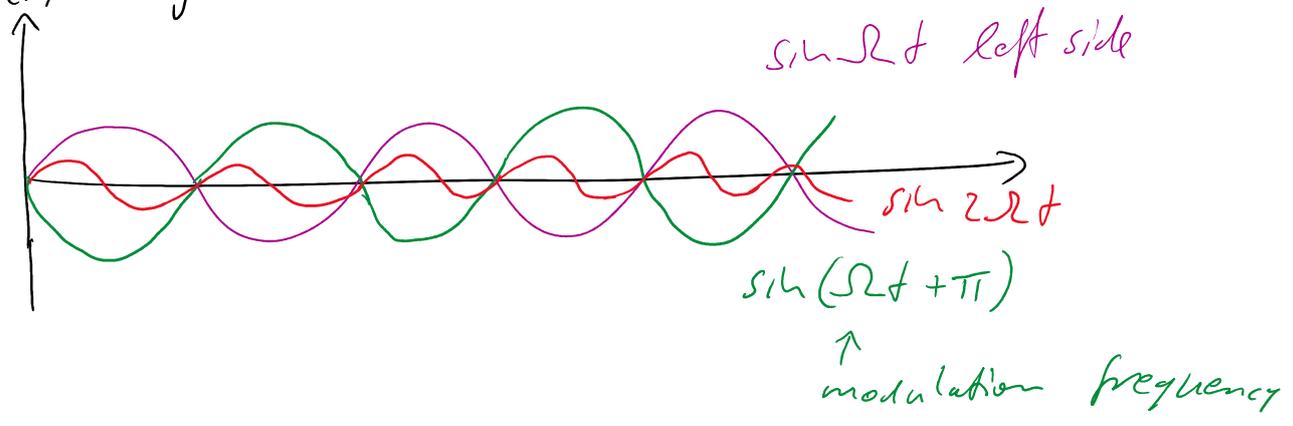
Lock-In Amplifier = phase-sensitive detector

$$\text{Laser } \vec{E}(t) = E_0 \sin(\omega_0 t + \sin \Omega t)$$

$$\omega(t) = \omega_0 + \sin \Omega t$$

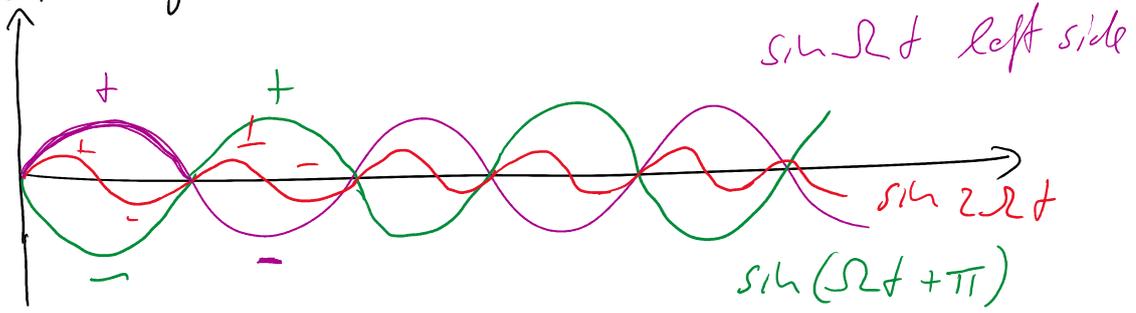


$$I(t) = \text{Signal}$$

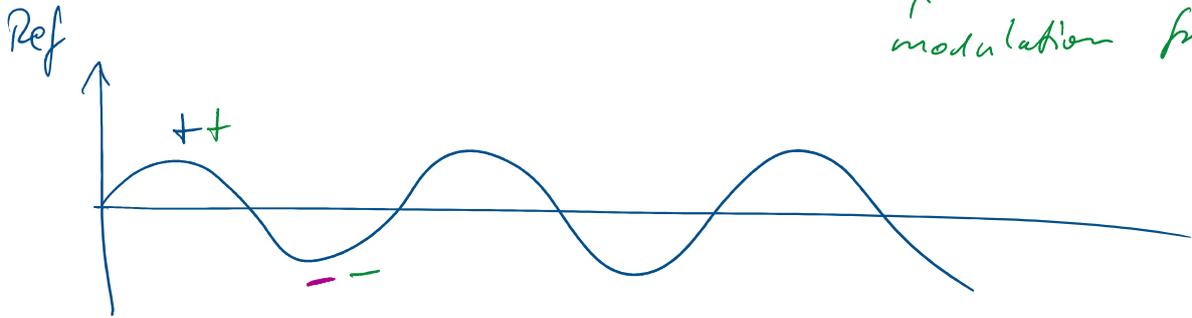


$$I(t) = \text{Signal}$$

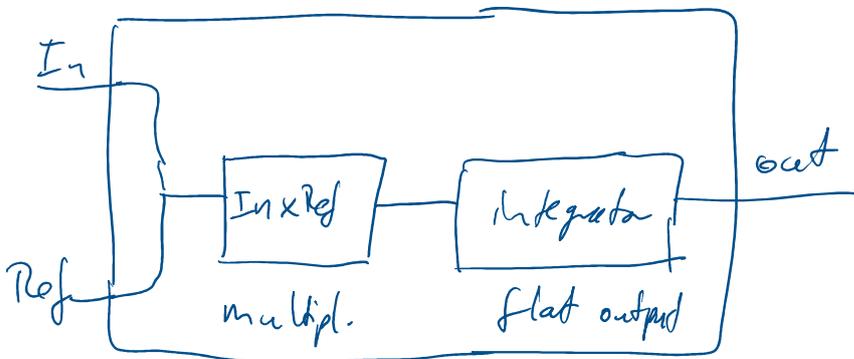
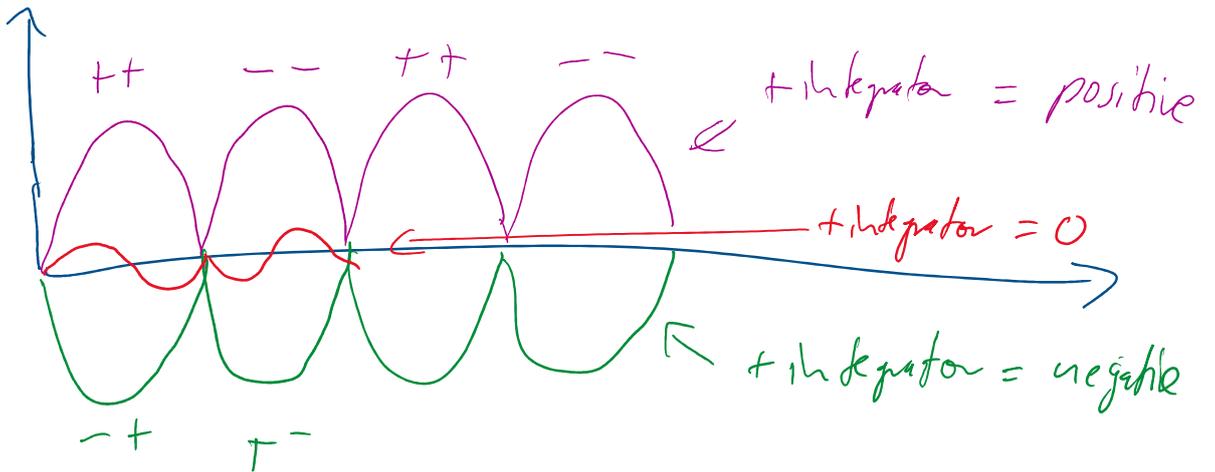
$I(t) = \text{Signal}$



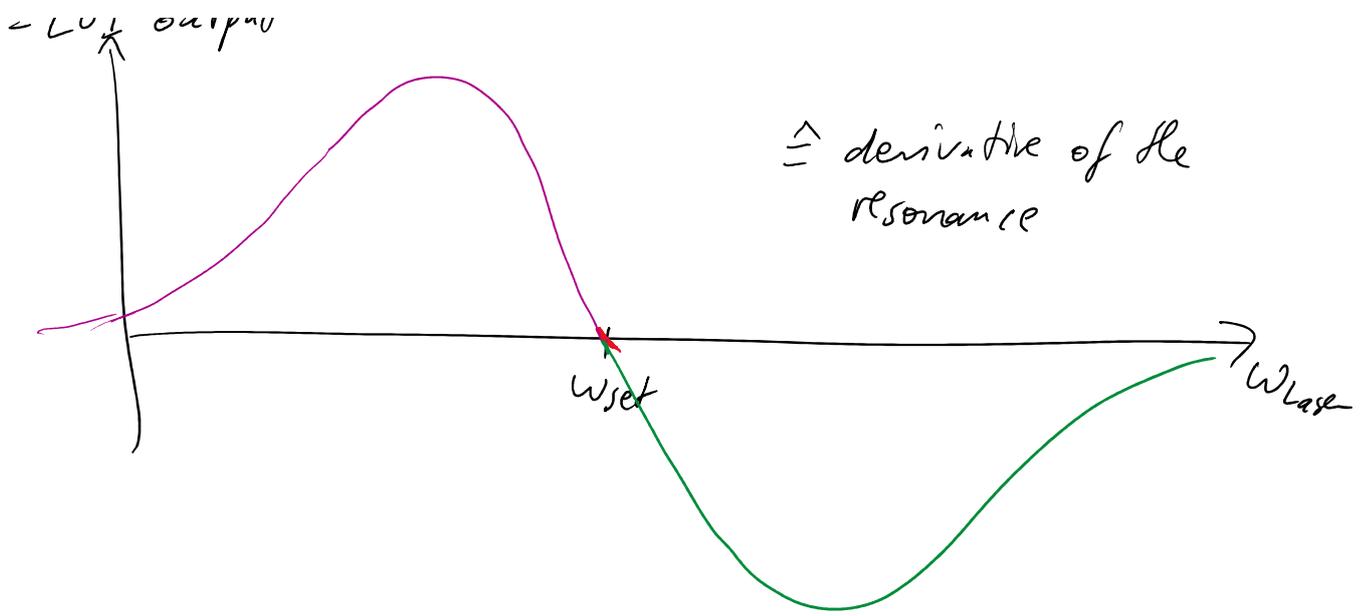
↑ modulation frequency



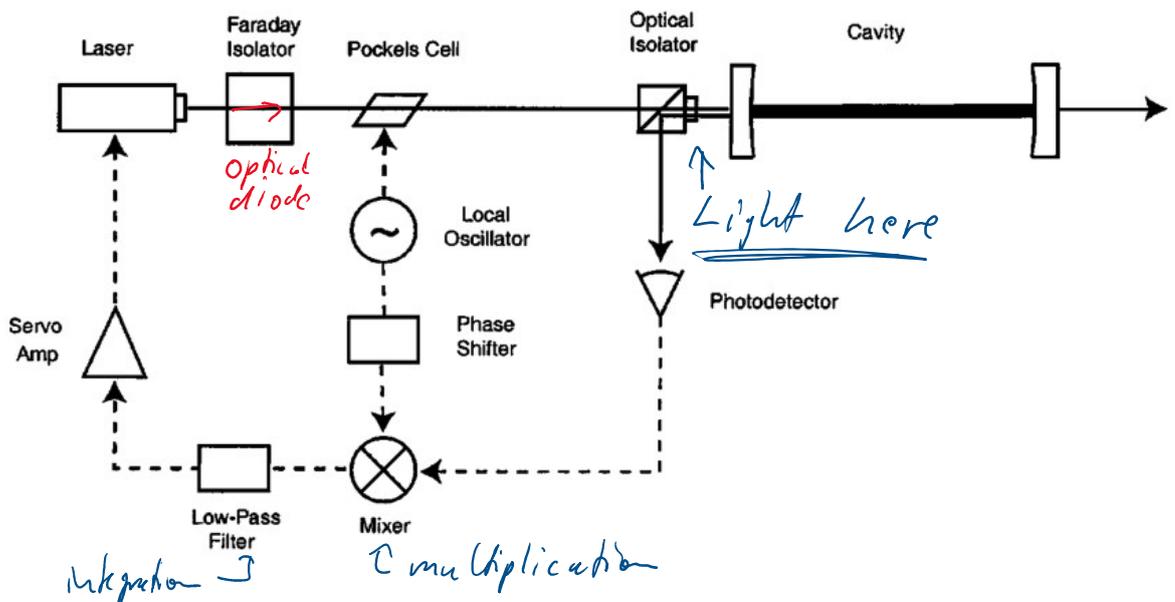
out = Signal x Ref.



Error signal =  $LO \bar{I}$  output



Pound-Drever-Hall PDH Lock  
 Black, Am. J. Phys. (2001)



$$E_{inc} = E_0 e^{i\omega t}$$

$$E_{refl} = E_1 e^{i\omega t}$$

$E_0, E_1 = \text{complex}$

$\downarrow$   
 they have a phase

reflection coefficient

$$F(\omega) = \frac{E_{refl}}{E_{inc}} = \frac{r \left( \exp\left(i \frac{\omega}{4\nu_{FSR}}\right) - 1 \right)}{1 - r^2 \exp\left(i \frac{\omega}{4\nu_{FSR}}\right)}$$

$$\bar{E}_{inc} \quad 1 - r^2 \exp\left(i \frac{\omega}{\Delta \nu_{FSR}}\right)$$

$r =$  Amplitude reflector of mirror

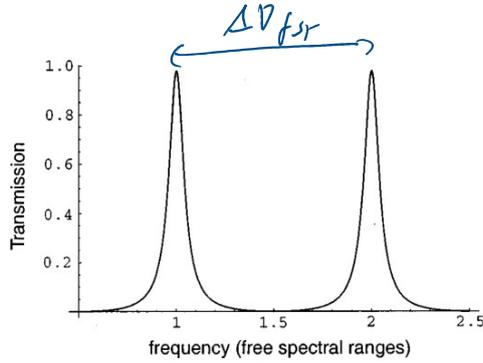
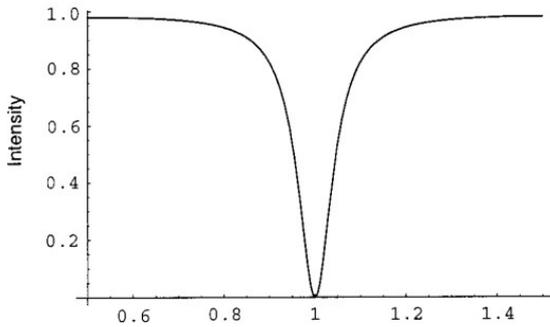


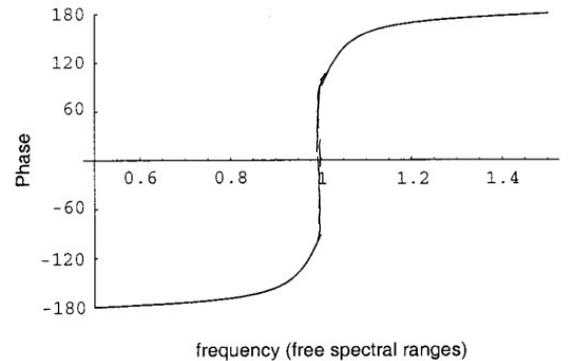
Fig. 1. Transmission of a Fabry-Perot cavity vs frequency of the incident light. This cavity has a fairly low finesse, about 12, to make the structure of the transmission lines easy to see.

Expl. = coherent sum of directly reflected incoming light and light leaking out of the cavity

$F(\omega)$



real part



Im. part

$$\bar{E}_{inc} = \bar{E}_0 e^{i(\omega t + \beta \sin \Omega t)}$$

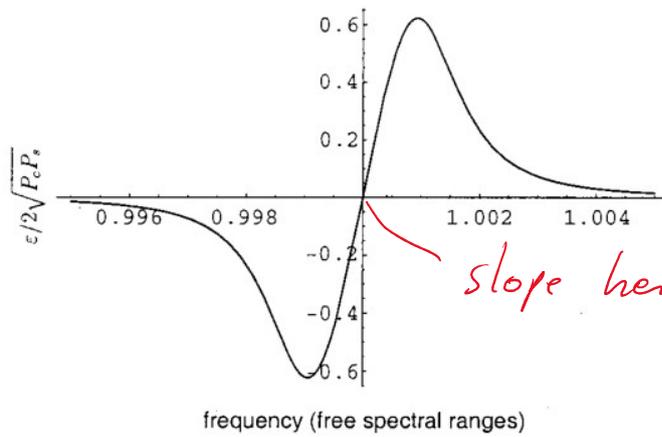
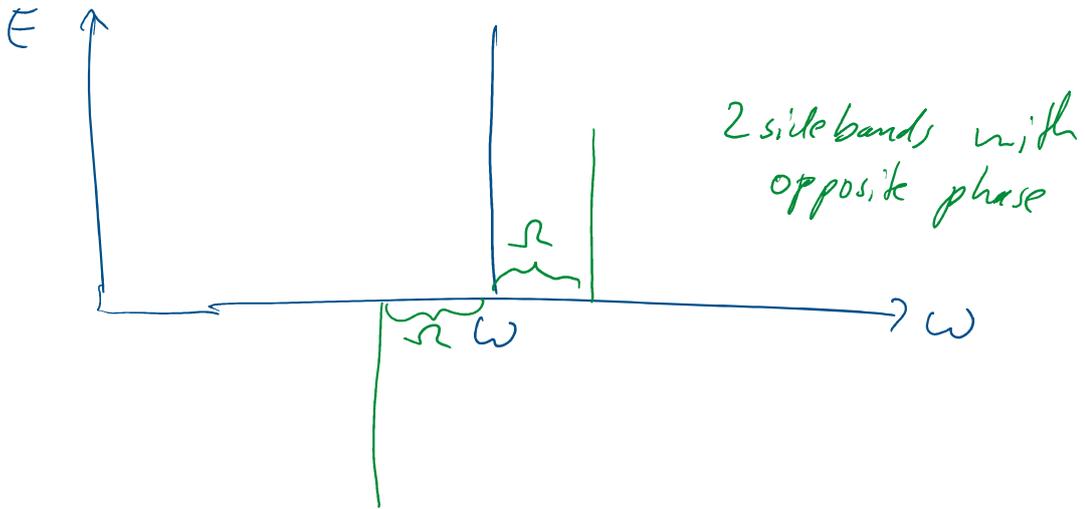
↙ phase change by Pockels cell

↑ modulation index

$$\approx \left[ \bar{E}_0(B) + 2i \bar{E}_1(B) \sin \Omega t \right] e^{i\omega t}$$

$$\approx L f_0(\beta) + 2i f_1(\beta) \sin JL + J e$$

Bessel



slope here defines the quality of the laser lock

Fig. 6. The Pound-Drever-Hall error signal,  $\epsilon/2\sqrt{P_c P_s}$  vs  $\omega/\Delta\nu_{\text{fsr}}$ , when the modulation frequency is low. The modulation frequency is about half a linewidth: about  $10^{-3}$  of a free spectral range, with a cavity finesse of 500.

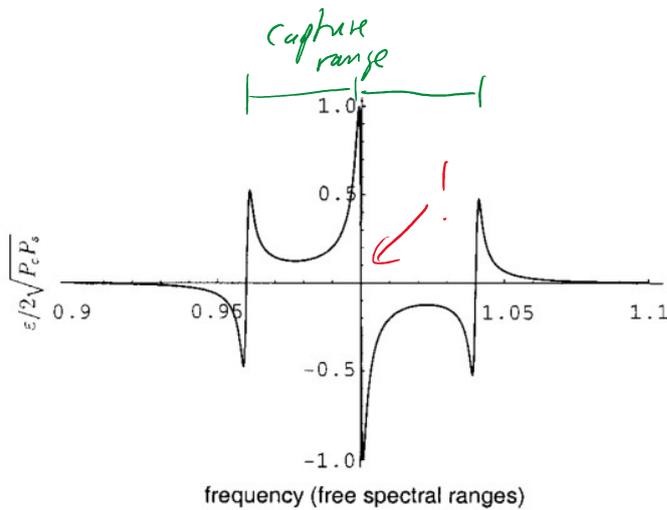


Fig. 7. The Pound-Drever-Hall error signal,  $\epsilon/2\sqrt{P_c P_s}$  vs  $\omega/\Delta\nu_{\text{fsr}}$ , when the modulation frequency is high. Here, the modulation frequency is about 20 linewidths: roughly 4% of a free spectral range, with a cavity finesse of 500.