→ Robert Boyd "Nonlinear Optics"

Maxwell's Equations:

$\nabla \underline{\tilde{D}} = \tilde{\rho}$ ← rapidly oscillating, e.g. $\tilde{E}(t) = E \cdot \cos \omega t$
↳ light frequency

$\nabla \underline{\tilde{B}} = 0$

$\nabla \times \underline{\tilde{E}} = -\frac{\partial}{\partial t} \underline{\tilde{B}}$

$\nabla \times \underline{\tilde{H}} = -\frac{\partial}{\partial t} \underline{\tilde{D}} + \tilde{j}$

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we will look <u>only</u> at dielectric materials with

$\tilde{\rho} = 0$ no free charges

$\tilde{j} = 0$ no free currents

$\underline{\tilde{B}} = \mu_0 \tilde{H}$ nonmagnetic materials

<u>but</u>

$\tilde{D} = \varepsilon_0 \tilde{E} + \tilde{P}$ <u>non</u> linear material

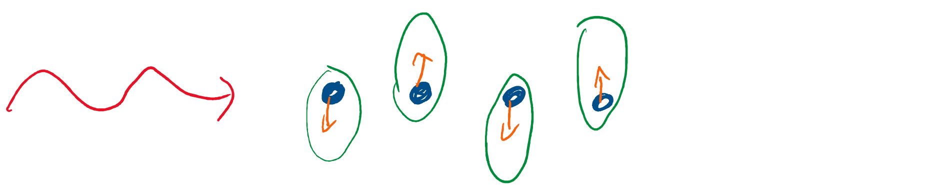
$\tilde{P} = \varepsilon_0 \chi \tilde{E}$ Polarization $\frac{\text{Dipole moment}}{\text{Volume}} = \frac{C}{m^2}$ (Cm, m^3)

The Dipole moment in the material is induced by the oscillating electric light field

susceptibility

refr. index

$\boxed{n^2 := 1 + \chi}$



refr. index

$$\boxed{n^2 := 1+\chi}$$

oscillating Dipole moment

curl of 3rd Maxwell

$$\vec{\nabla}\times(\vec{\nabla}\times\tilde{E}) = \vec{\nabla}\times\left(-\frac{\partial}{\partial t}\tilde{B}\right) =$$ interchange $\vec{\nabla}\times$ and $\partial/\partial t$

$$= -\frac{\partial}{\partial t}(\vec{\nabla}\times\tilde{B})$$

$\mu_0\tilde{H}$ 4th Maxwell

$$= -\mu_0\frac{\partial}{\partial t}(\vec{\nabla}\times\tilde{H}) =$$

$$= -\mu_0\frac{\partial^2}{\partial t^2}\tilde{D} + 0$$

$\vec{j}=0$ no currents

$$\vec{\nabla}\times\vec{\nabla}\times\tilde{E} + \mu_0\frac{\partial^2}{\partial t^2}\tilde{D} = 0$$

wave equn. inside material

with $\tilde{D} = \varepsilon_0\tilde{E} + \tilde{P}$ we get rid of $\tilde{D}$

$$\boxed{\vec{\nabla}\times\vec{\nabla}\times\tilde{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\tilde{E} = -\frac{1}{\varepsilon_0 c^2}\frac{\partial^2}{\partial t^2}\tilde{P}}$$

most general form of wave equation in NLO

$$\vec{\nabla}\times\vec{\nabla}\times\tilde{E} = \vec{\nabla}(\vec{\nabla}\tilde{E}) - \nabla^2\tilde{E}$$

$=0$ because we look at source-free media

free no charges

$$\nabla\tilde{D} = 0 \Leftrightarrow \nabla\tilde{E} = 0$$

$$\boxed{\nabla^2 \tilde{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\tilde{E} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2}{\partial t^2}\tilde{P}}$$

$$\tilde{P}(t) = \varepsilon_0\left[\chi^{(1)}\tilde{E}(t) + \chi^{(2)}\tilde{E}^2(t) + \chi^{(3)}\tilde{E}^3(t) + \ldots\right]$$

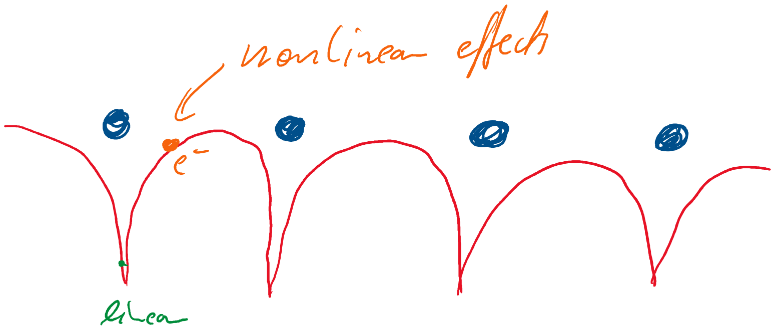
$$= \underbrace{\tilde{P}^{(1)}(t)}_{\text{linear optics}} + \underbrace{\tilde{P}^{(2)}(t) + \tilde{P}^{(3)}(t) + \ldots}_{\tilde{P}^{NL}(t)}$$

Non linear Optics

These non linear terms become important when $\tilde{E}(t) \approx E_{Atom} \approx \dfrac{e^2}{4\pi\varepsilon_0 a_0^2} \approx 6\cdot 10^{11}\,\frac{V}{m}$

Bohr radius

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m e^2}$$



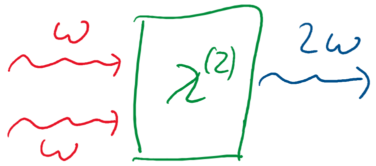
$$\tilde{P}^{(2)} \approx \tilde{P}^{(1)} \rightarrow \chi^{(2)} \approx \frac{\chi^{(1)}}{E_{atom}} \approx 1.6\cdot 10^{-12}\,\frac{m}{V}$$

$$\tilde{P}^{(3)} \rightarrow \chi^{(3)} \approx \frac{\chi^{(2)}}{E_{atom}} \approx 2.6\cdot 10^{-24}\,\frac{m^2}{V^2}$$

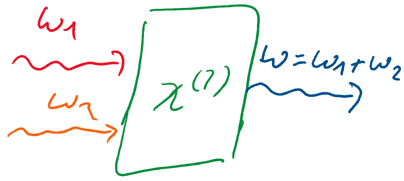
actually good approx. for most materials

$\chi^{(1)}$: ε, n

$\chi^{(2)}$: SHG 2nd harmonic generation



SFG sum frequency gen.



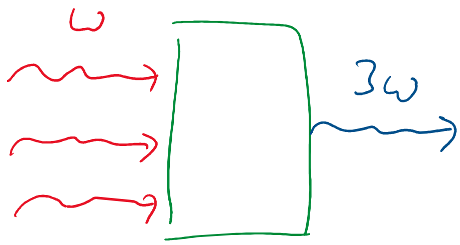
DFG difference ....

OR optical rectification

OPO

⋮

$\chi^{(3)}$: THG



intensity-dependent refractive index

4 wave mixing