Nonlinear wave equation:

$$\nabla^2 \tilde{E} - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\tilde{E} = \mu_0 \frac{\partial^2}{\partial t^2}\tilde{P}^{NL}$$

$$\tilde{E}(t) = E_0 \cos(\omega t - kz)$$

spatial variation

$$= E_0\left(e^{-i\omega t} + e^{+i\omega t}\right)$$

$$= E_0 e^{-i\omega t} + c.c.$$

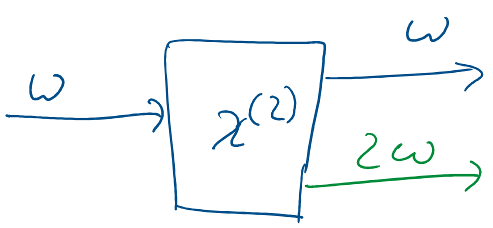
SHG

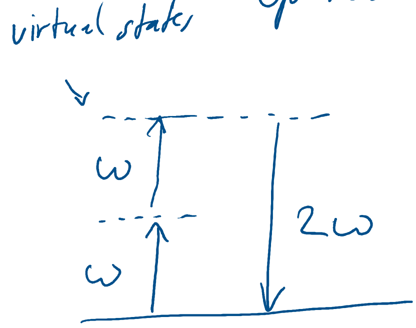
$$\tilde{P}^{NL}(t) = \tilde{P}^{(2)}(t) = \varepsilon_0 \chi^{(2)} \tilde{E}^2(t)$$

$$= \varepsilon_0 \chi^{(2)} \Big[\underbrace{E^2 e^{-2i\omega t} + E^{*2} e^{2i\omega t}}_{\text{polarization response at } 2\omega} + \underbrace{2EE^*}_{\text{constant}}\Big]$$

$\Rightarrow$ emits radiation at 2ω

optical rectification

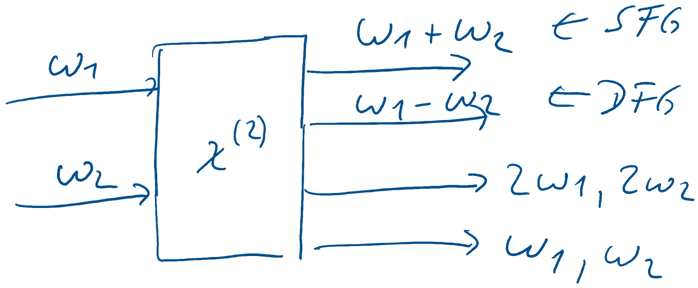




SFG + DFG

SFG + DFG

sum- & difference frequency generation



not simultaneously
→ phase matching

$$\tilde{E} = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} + c.c.$$

$$\tilde{P} = \varepsilon_0 \chi^{(2)} \tilde{E}^2$$

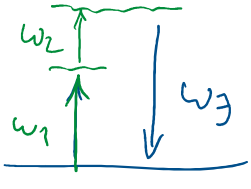
$$= \varepsilon_0 \chi^{(2)} \Big[E_1^2 e^{-2i\omega_1 t} + E_2^2 e^{-2i\omega_2 t} + \qquad \text{SHG}$$

$$+ 2E_1E_2\, e^{i(\omega_1+\omega_2)t} + \qquad \text{SFG}$$

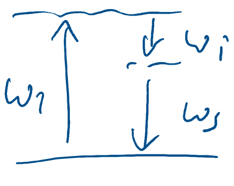
$$+ 2E_1E_2^*\, e^{i(\omega_1-\omega_2)t} + c.c. \Big] \qquad \text{DFG}$$

$$+ 2\varepsilon_0 \chi^{(2)} \left[E_1E_1^* + E_2E_2^* \right]$$

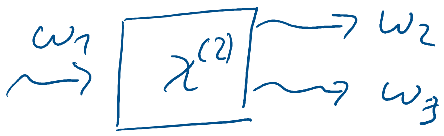
SFG



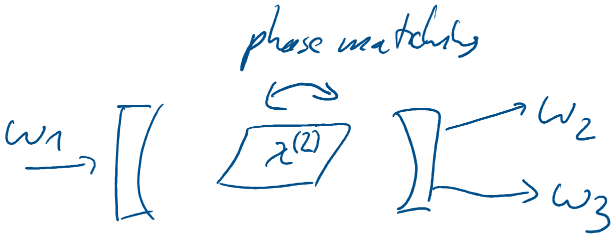
DFG

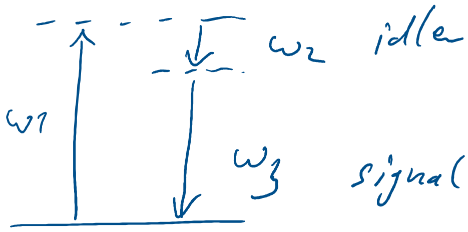


OPO Optical Parametric Oscillator



$$\omega_1 = \omega_2 + \omega_3$$

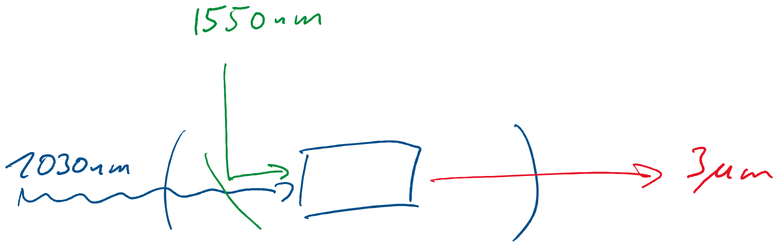




ω_2, ω_3 "signal", "idler"

convention $\omega_s > \omega_i$

e.g.



$$\frac{1}{\lambda_3} = \frac{1}{1030\,\text{nm}} - \frac{1}{1550\,\text{nm}}$$

$\boxed{\text{Phase matching}}$ decides if / which process is efficient

Boyd Chapter 2.2

coupled-wave equations for sum-frequency SFG

$$-\nabla^2 \tilde{E} + \frac{\varepsilon^{(1)}}{c^2}\frac{\partial^2 \tilde{E}}{\partial t^2} = -\frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \tilde{P}^{NL}}{\partial t^2} \qquad (2.1.17)$$

$$\tilde{E}(\vec{r},t) = \sum_n{}' \tilde{E}_n(\vec{r},t)$$

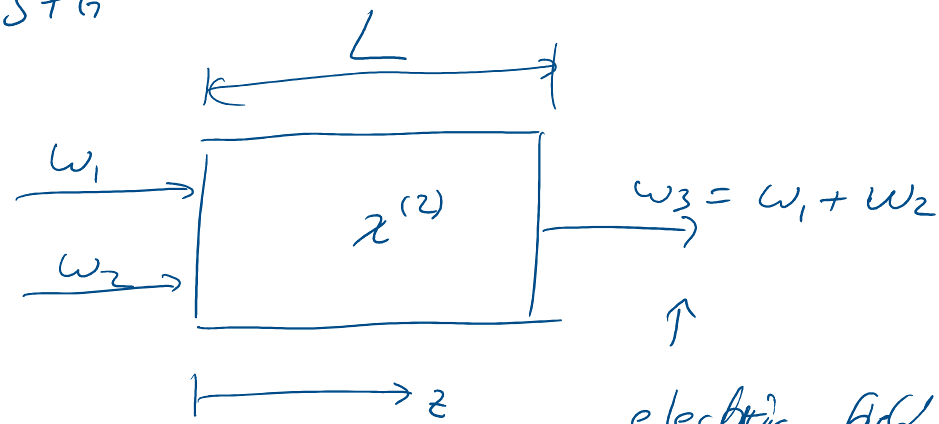
at $\omega_1, \omega_2, 2\omega_1, 2\omega_2, \omega_1+\omega_2, |\omega_1-\omega_2|$

$$\tilde{P}^{NL}(\vec{r},t) = \sum{}' \tilde{P}^{NL}(\vec{r},t)$$

$$\tilde{P}^{NL}(\vec{r},t) = \sum_n \tilde{P}_n^{NL}(\vec{r},t)$$

$\omega_1 + \omega_2$, $|\omega_1 - \omega_2|$

e.g. SFG



electric field E_3 at $\omega_1 + \omega_2$

$$\tilde{E}_3(z,t) = A_3 e^{i(k_3 z - \omega_3 t)} + c.c. \qquad (2.2.1)$$

$$k_3 = \frac{n_3 \omega_3}{c}, \quad n_3^2 = \epsilon^{(1)}(\omega_3)$$

mildly efficient process: A_3 is slowly varying with z

$$\tilde{P}_3(z,t) = P_3 e^{-i\omega_3 t} + c.c. \qquad (2.2.3)$$

$$P_3 = 4 \varepsilon_0 d_{eff} E_1 E_2$$

effective nonlinearity

$$d_{eff} = \frac{1}{2} \chi^{(2)}$$

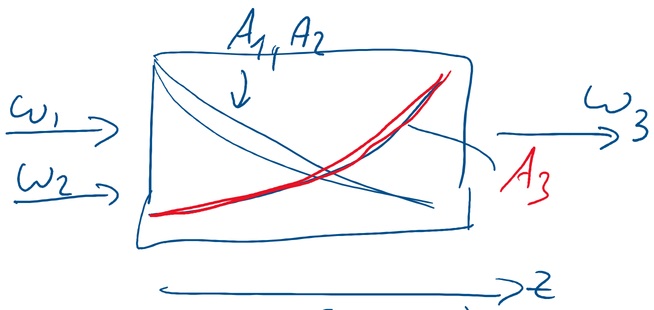
incoming fields $\tilde{E}_i(z,t) = E_i e^{-i\omega_i t} + c.c.$ ← temporal

$i = 1, 2$

where $E_i = A_i e^{i k_i z}$ ← slow

where $E_i = A_i e^{ik_i z}$ ← slow spatial variation

what we want



E_1, E_2, E_3 into (2.1.17)

$$\nabla^2 \longrightarrow \frac{d^2}{dz^2} \qquad \text{(plane waves)}$$

$$\left[\frac{d^2 A_3}{dz^2} + 2ik_3 \frac{dA_3}{dz} \underbrace{- k A_3 + \frac{\epsilon^{(1)}(\omega_3)\omega_3^2 A_3}{c^2}}_{\text{cancel} \Rightarrow \; = 0}\right] e^{i(k_3 z - \omega_3 t)} + c.c.$$

$$= \frac{-4 d_{eff}\, \omega_3^2}{c^2} A_1 A_2 e^{i[(k_1+k_2)z - \omega_3 t]} + c.c.$$

$$\frac{d^2 A_3}{dz^2} + 2ik_3 \frac{dA_3}{dz} = -\frac{4 d_{eff}\, \omega_3^2}{c^2} A_1 A_2 e^{i(k_1+k_2-k_3)z}$$

neglect, because it is typically very small

⇒ slowly-varying amplitude approx.

$$\frac{dA_3}{dz} = \frac{2i\, d_{eff}\, \omega_3^2}{k_3 c^2} A_1 A_2 e^{i\Delta k z}$$

$\Delta k = k_1 + k_2 - k_3$ wave vector mismatch

$$\frac{dA_1}{dz} = \frac{2i\, d_{eff}\, \omega_1^2}{k_1 c^2} A_3 A_2^* e^{-i\Delta k z}$$

$$\frac{dA_2}{dz} = \quad \ldots\ldots$$

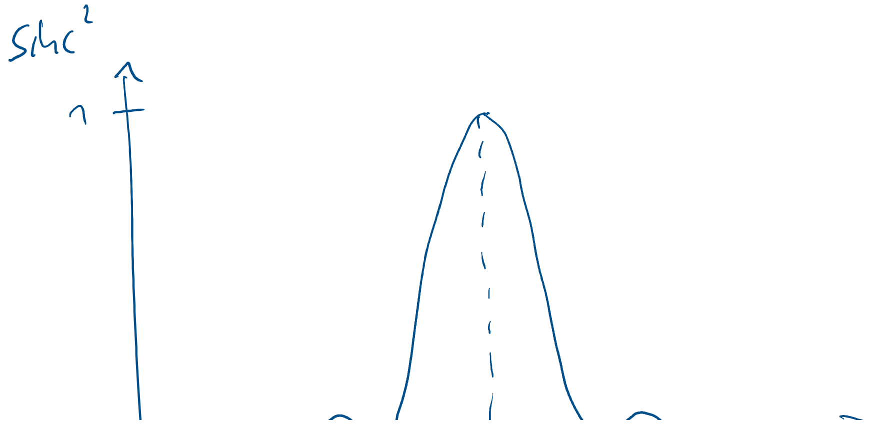
depletion of incoming light

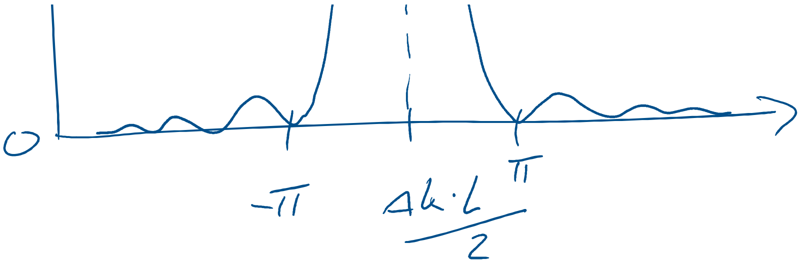
$$\boxed{\frac{dA_3}{dz} = \frac{2i\, d_{eff}\, \omega_3^2}{k_3 c^2} A_1 A_2 e^{i\Delta k z}}$$

if $\Delta k = 0$ then $e^{i\Delta k z} = 1$

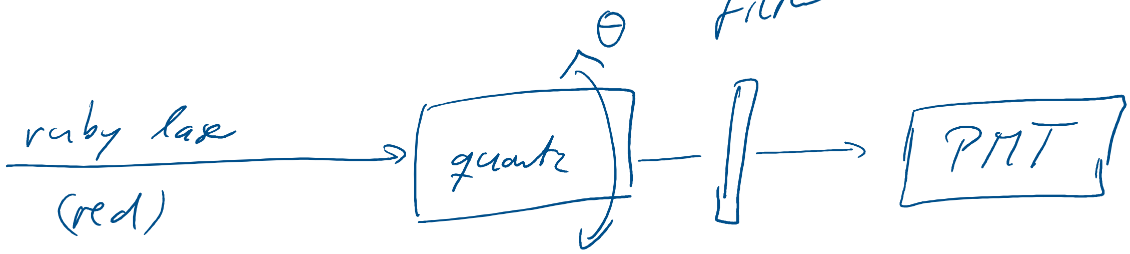
A_3 is increasing linearly with z

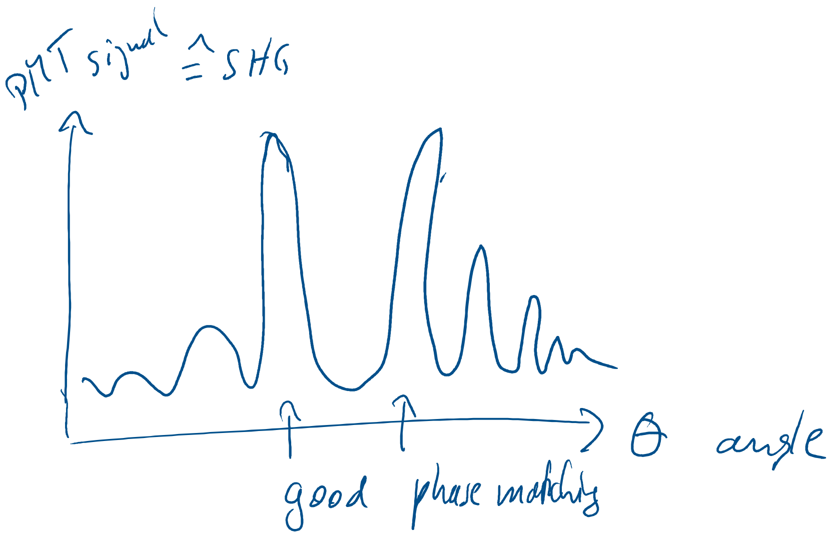
intensity $$I_3 = \frac{8\, d_{eff}^2\, \omega_3^2\, I_1 I_2}{n_1 n_2 n_3 \varepsilon_0 c^2} L^2 \operatorname{sinc}^2\left(\frac{\Delta k L}{2}\right)$$ (2.2.19)





Maker et al. (1962)

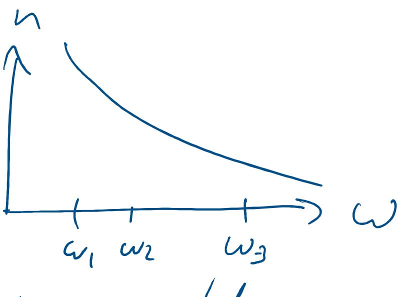




good phase matching is not trivial

$\omega_1 \leq \omega_2 \leq \omega_3$ SHG, SFG

normal dispersion n decreases with frequency



phase-matching would mean

phase-matching would mean

$$\frac{n_1 \omega_1}{c} + \frac{n_2 \omega_2}{c} = \frac{n_3 \omega_3}{c}$$

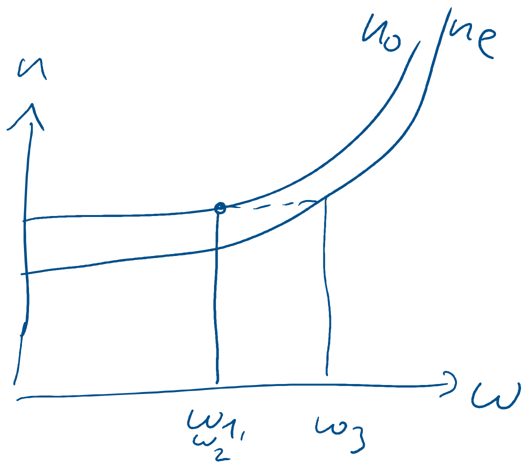
impossible

but what is possible:

negative uniaxial crystal

birefringent

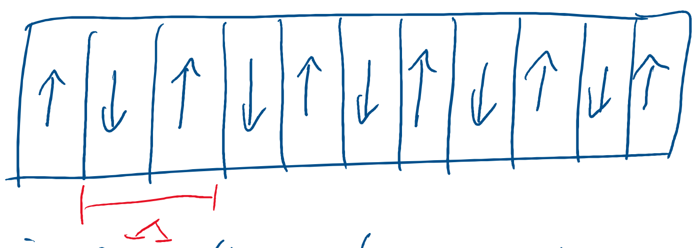
$\Rightarrow$ different polarizations can create phase matching



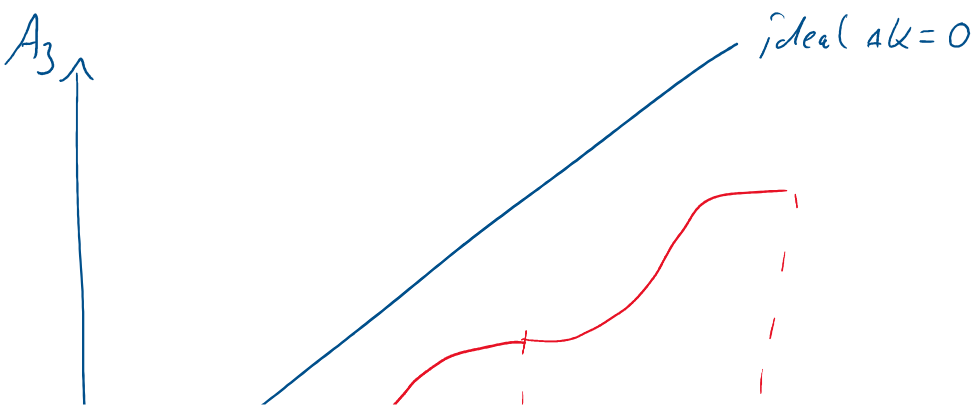
(angle, temperature tuning)

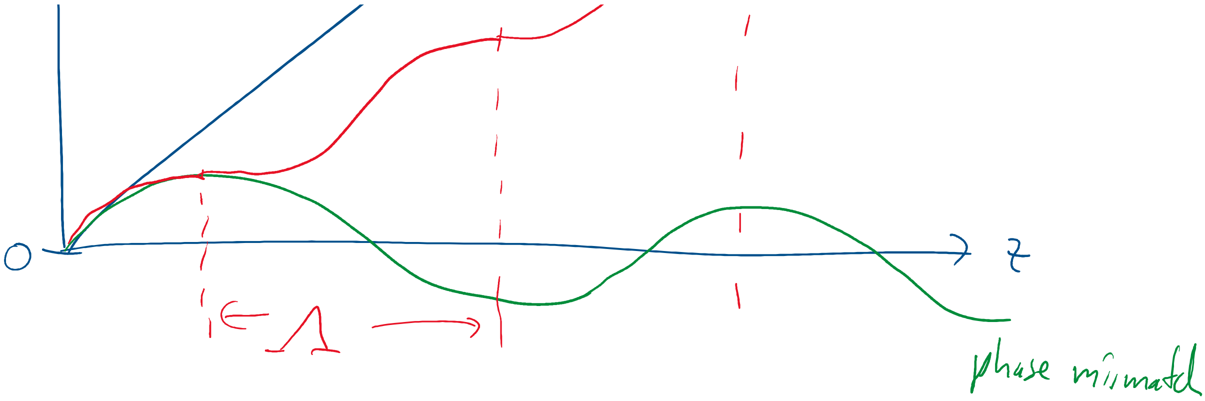
Today: periodically-poled nonlinear crystals

$\Rightarrow$ quasi phase-matching



periodic poling of crystal axis





e.g. PPLN

periodically poled $LiNbO_3$

Lithium Niobate