Next week's lectures start approx. 30 minutes later: 12:45

PREN — Proton Radius European Network

μASTI — μ Atom Spectroscopy Theory Initiative

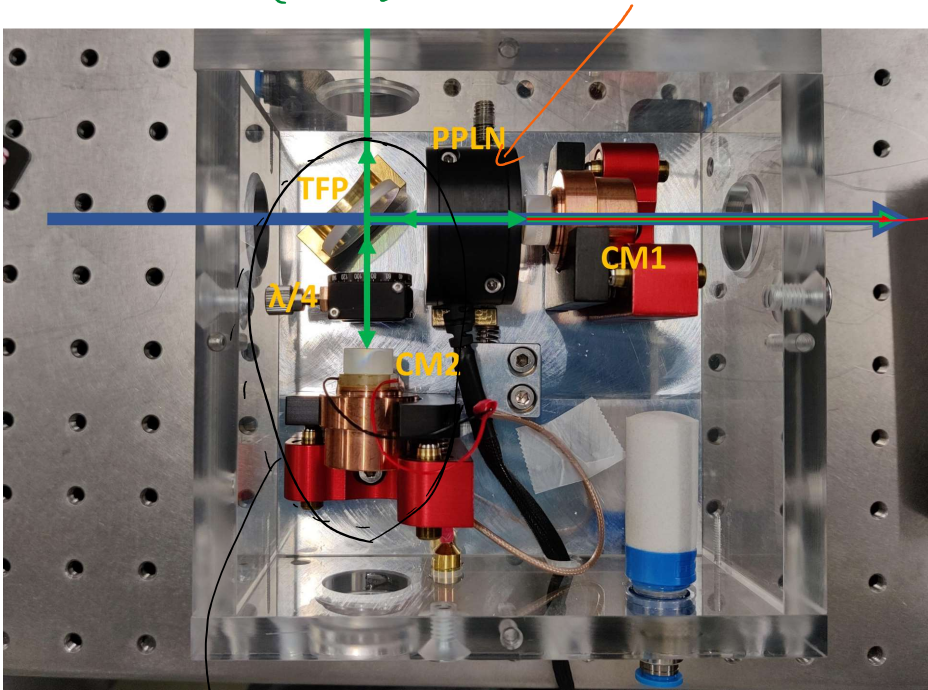
Workshop @ HIM

Our OPO in real life

tunable
cw 1550nm

PPLN = OPO crystal
in oven

pulsed
1030nm
fixed λ



3 μm

$$\frac{1}{3000\,\text{nm}} = \frac{1}{1030\,\text{nm}} - \frac{1}{1550\,\text{nm}}$$

≙ tuned

tune

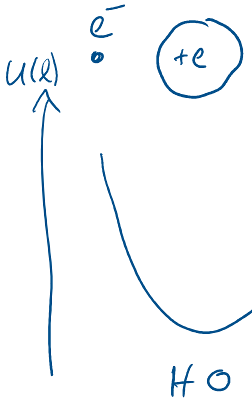
variable incoupler based on polarization

Nonlinear susceptibility $\chi^{(2)}, \chi^{(3)}$ of a classical anharmonic oscillator

Boyd ch. 1.4.

Lorenz-model of atom ≙ e^- in harmonic oscillator

$u(x)$ e^- $(+e)$

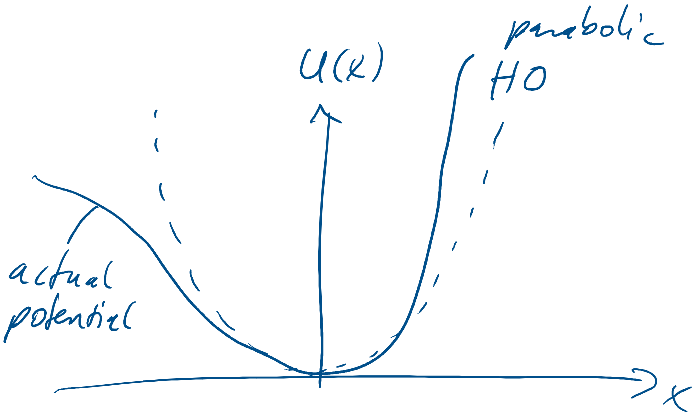


parabolic $\hat{=}$ HO

anharmonic oscillator:

① no inversion symmetry

non-centrosymmetric materials



equation of motion

$$\ddot{\tilde{x}} + 2\gamma\dot{\tilde{x}} + \underbrace{\omega_0^2\tilde{x} + a\tilde{x}^2}_{\text{restoring force} = F_{\text{restoring}}} = -\frac{e\tilde{E}(t)}{m}$$

damping — restoring force $= F_{\text{restoring}}$ — external force (driven oscillator

a = strength of (anharmonic) nonlinearity

$$U(\tilde{x}) = -\int F_{\text{restoring}}\, d\tilde{x} = \frac{1}{2} m\omega_0^2\tilde{x}^2 + \underline{\frac{1}{3} m a \tilde{x}^3}$$

applied field: $\tilde{E}(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} + c.c.$

perturbation theory

if $a = 0$ → HO solutions for very weak field

$E(t)$ is small

if $a = 0 \Rightarrow$ the solution [illegible]

a large increase nonlinearity $\qquad E(t)$ is small

expansion parameter λ

varying λ varies the strength of the applied field

$$\tilde{E}(t) \longmapsto \lambda \tilde{E}(t)$$

$$\ddot{\tilde{x}} + 2\gamma \dot{\tilde{x}} + \omega_0^2 \tilde{x} + a\tilde{x}^2 = -\lambda \tilde{E}(t) \frac{e}{m}$$

search for a solution

$$\tilde{x} = \lambda \cdot \tilde{x}^{(1)} + \lambda^2 \tilde{x}^{(2)} + \lambda^3 \tilde{x}^{(3)} + \ldots\ldots$$

solution should exist for any value of λ

$\Rightarrow$ sort parts according to power in λ

$$\lambda: \quad \lambda \ddot{\tilde{x}}^{(1)} + 2\gamma \lambda \dot{\tilde{x}}^{(1)} + \omega_0^2 \lambda \tilde{x}^{(1)} + 0 = -\lambda \tilde{E}(t) \frac{e}{m} \leftarrow HO$$

$$\lambda^2: \quad \ddot{\tilde{x}}^{(2)} + 2\gamma \dot{\tilde{x}}^{(2)} + \omega_0^2 \tilde{x}^{(2)} + a\left[\tilde{x}^{(1)}\right]^2 = 0 \leftarrow$$

$$\lambda^3: \quad \ddot{\tilde{x}}^{(3)} + 2\gamma \dot{\tilde{x}}^{(3)} + \omega_0^2 \tilde{x}^{(3)} + 2a \tilde{x}^{(1)} \tilde{x}^{(2)} = 0$$

$$\vdots$$

λ term $\hat{=}\ x^{(1)}\ \hat{=}\ HO$

$$\Rightarrow x^{(1)}(t) = x^{(1)}(\omega_1) e^{-i\omega_1 t} + x^{(2)}(\omega_2) e^{-i\omega_2 t} + c.c.$$

amplitudes
$$x^{(1)}(\omega_j) = -\frac{e}{m} \frac{E_j}{D(\omega_j)}$$

"denominator function"

$D(\omega_j)$ "denominator function"

$$D(\omega_j) = \omega_0^2 - \omega_j^2 - 2i\omega_j\gamma$$

now <u>square</u> $x^{(1)}(t)$ and put it into χ^2 equation

$\hookrightarrow$ gives us $\pm 2\omega_1$, $\pm 2\omega_2$, $\pm(\omega_1+\omega_2)$, $\pm(\omega_1-\omega_2)$, 0

e.g. response at $2\omega_1$ is obtained from solving:

$$\ddot{\tilde{x}}^{(2)} + 2\gamma\dot{\tilde{x}}^{(2)} + \omega_0^2\tilde{x}^{(2)} = \frac{-a(eE_1/m)^2 e^{-i\cdot 2\omega_1 t}}{D^2(\omega_1)}$$

look at steady state solution like:

$$\tilde{x}^{(2)}(t) = x^{(2)}(2\omega_1)\, e^{-2i\omega_1 t}$$

$\Rightarrow$ result

$$x^{(2)}(2\omega_1) = \frac{-a\left(\frac{e}{m}\right)^2 E_1^2}{D(2\omega_1)\cdot D^2(\omega_1)}$$

$\vdots$

$$x^{(2)}(\omega_1-\omega_2) = \frac{-a\left(\frac{e}{m}\right)^2 E_1 E_2^*}{D(\omega_1-\omega_2)\, D(\omega_1)\, D(-\omega_2)}$$

simple model $\to$ reality

$\chi^{(1)}$, $\chi^{(2)}$ susceptibilities

lin:

lin:

$$P^{(1)}(\omega_j) = \varepsilon_0 \chi^{(1)}_{(\omega_j)} E(\omega_j)$$

$$P^{(1)}(\omega_j) = - N \cdot e \cdot x^{(1)}(\omega_j)$$

number density of atoms

$$\Rightarrow \chi^{(1)}_{(\omega_j)} = \frac{N \cdot e^2/m}{\varepsilon_0 \, D(\omega_j)}$$

NL, e.g. SHG

$$P^{(2)}(2\omega_1) = \varepsilon_0 \chi^{(2)}(\overset{\text{out}}{2\omega_1}, \overset{\text{in}}{\omega_1}, \overset{\text{in}}{\omega_1}) \, E^2(\omega_1)$$

$$P^{(2)}(2\omega_1) = N e x^{(2)}(2\omega_1)$$

$$\chi^{(2)}(2\omega_1, \omega_1, \omega_1) = \frac{N \left(\frac{e^3}{m^2}\right) \cdot a}{\varepsilon_0 \, D(2\omega_1) \, D^2(\omega_1)}$$