Light polarization & the Jones formalism

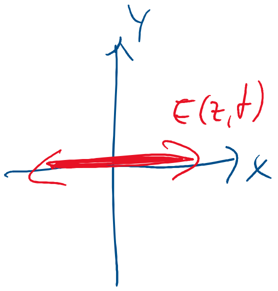
Dienstag, 4. Juli 2023   12:25

Electromagnetic waves → transverse waves

Polarization refers to the direction of the electric field vector

$$\vec{E}(z,t) = \begin{pmatrix} E_x \cos(\omega t - kz) \\ E_y \cos(\omega t - kz \pm \varphi) \\ 0 \end{pmatrix}$$

$\vec{H}(z,t)$ is always there and <u>in phase</u> with the $\vec{E}$ field



linear polarized light

$E_y = 0$

propagating wave

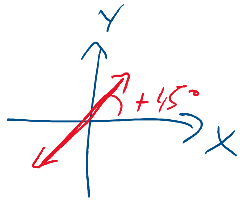
$E = E_x \cos(\omega t - kz)$

short-hand notation at $z=0$, $E = 1$

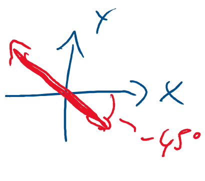
horizontal lh: $E = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

vertical lh: $E = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

+45° lh: $E = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$



−45° lh $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$



circular polarized light: phase difference between E_x & E_y

$\vec{\ } \quad (E_x(t)) \quad (A_x \, e^{i\delta_x})$

between E_x & E_y

$$\vec{E} = \begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix} = \begin{pmatrix} A_x \ e^{i\delta_x} \\ A_y \ e^{i\delta_y} \end{pmatrix}$$

Amplitude

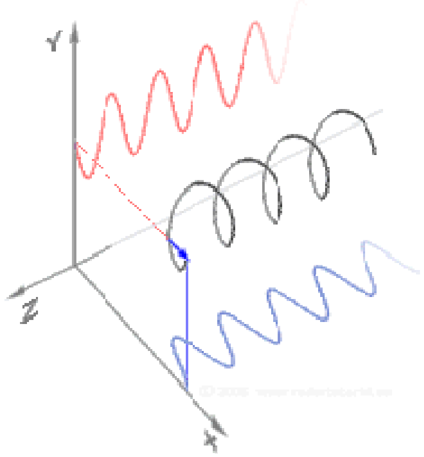
phase

$A_x = A_y$

$\delta_y - \delta_x = +90°$

$$\vec{E} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ +i \end{pmatrix}$$

right handed circular polarized light



→ animation on Wikipedia

$\delta_y - \delta_x = -90°$ → left-handed circular

$$\vec{E} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

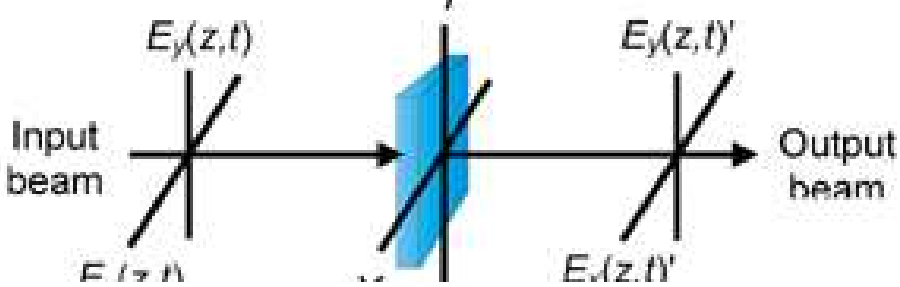
Jones vectors

Jones Calculus

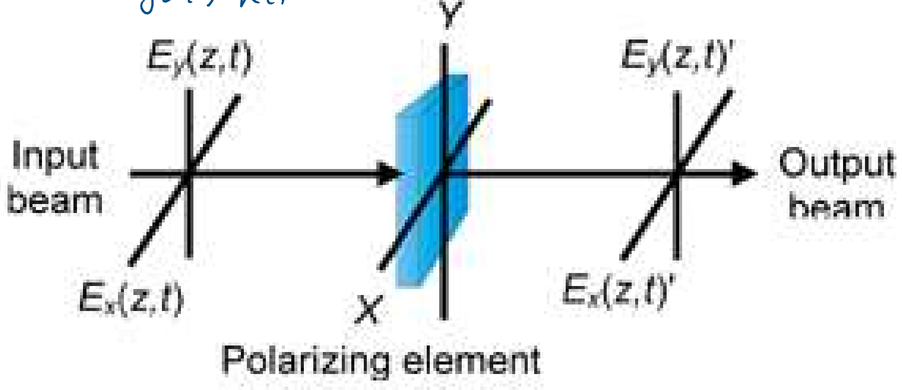
Jones vector

matrix

Jones vector



Jones vector matrix Jones vector



laser field
$$E = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} E_{0x}\, e^{i\delta_x} \\ E_{0y}\, e^{i\delta_y} \end{pmatrix}$$

intensity $= \quad I = \begin{pmatrix} E_x^* & E_y^* \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = E_x E_x^* + E_y E_y^*$

$$I = E^+ E$$

polarizing element $\quad J = \begin{pmatrix} j_{xx} & j_{xy} \\ j_{yx} & j_{yy} \end{pmatrix}$

e.g. horizontal linear polarizer $\quad J_{POL} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$$E_{out} = J \cdot E_{in}$$

$$\begin{pmatrix} E_x \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$E_x^{out} = \underline{j_{xx}} \cdot \underline{E_x} + \underline{j_{xy}} \cdot E_y$$

$$E_y^{out} = \underline{0} = \underline{j_{yx}} \cdot E_x + \underline{j_{yy}} \cdot E_y$$

lin vertical polarizer $\quad J_{POL-V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

lin $\pm 45°$ polarizer $\quad J = \frac{1}{2} \begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$

lin polarizer angle θ from horiz. $\quad J = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$

right / left circular polarizer

$$\frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$$

retarder plates $\left(\frac{\lambda}{2}, \frac{\lambda}{4} \text{ waveplates} \right)$

quarter-wave plates $\quad e^{\pm i \frac{\pi}{4}} \begin{pmatrix} 1 & 0 \\ 0 & \pm i \end{pmatrix}$ ← fast axis horizontal / vertical

–"– arbitrary fast axis angle

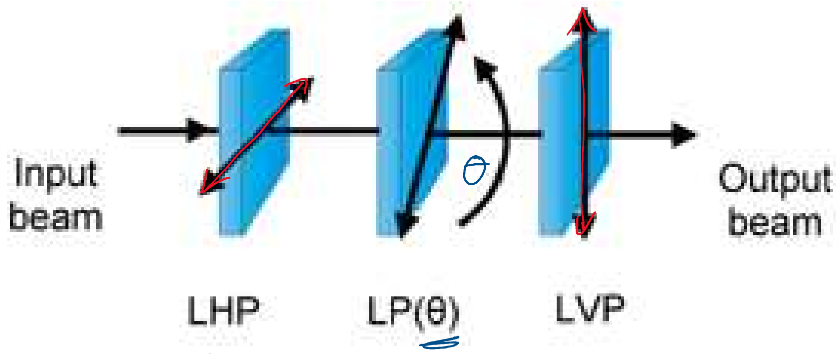
$$e^{-\frac{i\pi}{4}} \begin{pmatrix} \cos^2\theta + i\sin^2\theta & (1-i)\sin\theta\cos\theta \\ (1-i)\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix}$$

half-wave plate $\quad \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

θ = angle

example

example



lin pol. horizontal

lin polariser

vertical

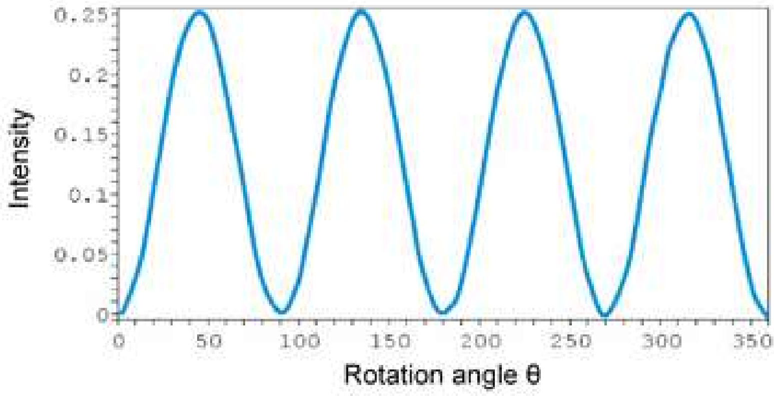
$$J_{tot} = J_{LVP} \cdot J_{LP}(\theta) \cdot J_{LHP} = \dots = \begin{pmatrix} 0 & 0 \\ \sin\theta\cos\theta & 0 \end{pmatrix}$$

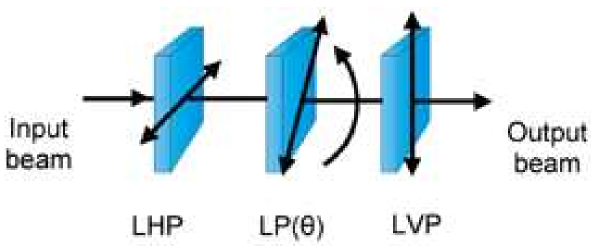
last

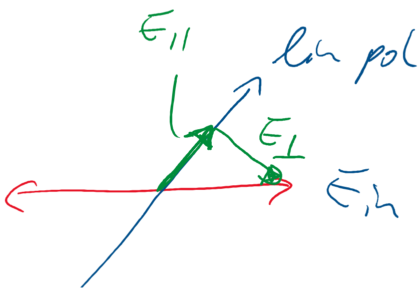
first element

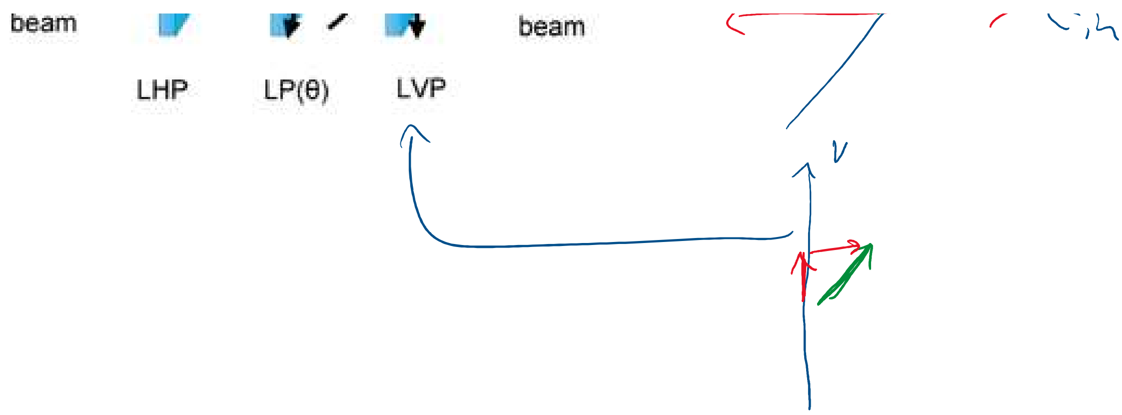
$$E_{out} = J_{tot} \cdot E_{in}$$

Intensity $I_{out} = E_{out}^{\dagger} E_{out} = \frac{1}{8}\left(1 - \cos 4\theta\right)$








beam
LHP
LP(θ)
LVP
beam